FPI

Financial Policy Institute
The Ministry of Finance of the Slovak Republic
www.finance.gov.sk/ifp

Output Gap and NAIRU Estimates within State-Space Framework: An Application to Slovakia

March 2010

Economic Analysis 16

Abstract

An accurate identification of the output gap, comprehensive concept for the measurement of the cyclical position of the economy, is necessary for the formulation of prudent macroeconomic policies. However, this indicator is unobservable in practice and therefore is subject to the estimation difficulties. For the purpose of the Ministry of Finance of the Slovak Republic, its estimates are used for the calculation of the cyclically-adjusted public finance deficit, which is required for the assessment of the stability and convergence programme. The MoFSR uses the conventional production method for the calculation of the output gap. Considering that significant FDI inflow in pre-crisis years injected volatility in the potential output growth, the MoFSR has included experts' adjustments to the production function methodology. In this paper, we aim to validate these adjustments by developing an alternative model for the output gap estimation. The multivariate Kalman filter framework is used as it secures flexible modeling environment and allows for simultaneous goods and labour markets interactions. As an additional product, we obtain the NAIRU estimates,which is an useful information for the formation and evaluation of labour market policies. We conclude that estimates derived from the model confirm our hypothesis that experts' adjustments to the standard production function are necessary to produce result in line with other observed economic indicators. Finally, by identifying the negative demand shocks from current economic and financial crisis, we try to evaluate the estimated loss on the potential output stemming from the global downturn. Alongside with this, we project a recovery of the potential output growth rate to its 'equilibrium' rates. The calculations support a widely shared consensus that the pace of the potential growth will decelerate compared with previous decade.

Authors:
Lucia Šrámková lucia.sramkova@mfsr.sk
Head of macroeconomic Analyses and Forecasts Department:
Martina Kobilicová martina.kobilicova@mfsr.sk
Director of FPI:
Zdenko Krajčír zdenko.krajcir@mfsr.sk

The author would like to thank Michal Horvath and Matus Senaj for interesting discussion and helpful suggestions. Invaluable assistance was provided by Milan Vanko. However, all remaining errors are the sole responsibility of the author. The views expressed in this paper are those of the author and do not necessarily represent those of the Ministry of Finance of the Slovak republic. Standard disclaimer applies.

Abstrakt

Presná identifikácia produkčnej medzery, ktorá predstavuje komplexný indikátor odhadu cyklického vývoja ekonomiky, je nevyhnutná pre formuláciu uvážlivých makroekonomických politík. Produkčná medzera je však štatisticky nemerateľná veličina a preto jej určenie podlieha ťažkostiam a chybám pri vyčíslení. Pre fiškálnu politiku určuje cyklicky očistený deficit verejných financií a charakter fiškálnej politiky, ktorý je používaný na vyhodnotenie Programu Stability SR. MFSR používa pre odhad produkčnej medzery metódu produkčnej funkcie, ktorá je aj referenčnou metódou EK. Vzhľadom na vysoký prílev priamych zahraničných investícií, ktoré viedli k zvýšenej volatilite rastu potenciálneho produktu, MFSR expertne upravuje výpočet rovnovážnej súhrnnej produktivity faktorov. V tomto článku, sa pokúsime expertné modifikácie podporiť alternatívnym modelom. Pre tento účel použijeme viacrozmerný Kalmanov filter, ktorý má flexibilnú štruktúru a umožňuje simultánne modelovanie trhu tovarov a služieb a trhu práce. Ďalším výsledkom analýzy je odhad miery nezamestnanosti neakcelerujúcej infláciu, ktorá je užitočným indikátorom pre formovanie a vyhodnocovanie politík trhu práce. Výsledky nášho modelu potvrdzujú opodstatnenosť expertných zásahov pri odhade produkčnej medzery v prípade Slovenska a zároveň získané odhady sú v súlade s ostatnými indikátormi nerovnováhy. V závere práce pomocou modelu kvantifikujeme kumulatívnu stratu na potenciálnom produkte, ktorá je dôsledkom súčasnej globálnej ekonomickej a finančnej krízy. V prípade Slovenska je pravdepodobné, že výpadok na úrovni potenciálneho produktu bude trvalý, avšak strata sa v priebehu nastávajúcej dekády stabilizuje. Predikcie modelu avšak tiež potvrdzujú široký konsenzus, že tempo rastu potenciálneho produktu sa v porovnaní s predkrízovými rokmi spomalí.

Autor:
Lucia Šrámková lucia.sramkova@mfsr.sk
Riaditeľka odboru makroekonomických analýz a prognóz:
Martina Kobilicová martina.kobilicova@mfsr.sk
Riaditeľ IFP:
Zdenko Krajčír zdenko.krajcir@mfsr.sk

Content

Non-Technical Summary

The assessment of the current state of the economy relies on the correct identification of the potential output and the output gap, the latter being a comprehensive measure of the cyclical position of the economy. However, both these theoretical concepts derived from the supply-side of the economy are unobservable in practice and therefore subject to the estimation difficulties and errors. Accurate estimates of these indicators are essential for the prudent fiscal and monetary policies. Therefore, intensive research has been conducted in this area and various quantitative methods have been used for the output gap estimation. For the purpose of the Ministry of Finance of the Slovak Republic (MoFSR), the estimates are used for the calculation of the cyclically-adjusted public finance deficit (structural primary public finance deficit) and the evaluation of the fiscal stance (pro- or counter-cyclical fiscal expansion/contraction). The former is necessary for the assessment of the stability and convergence programme, as posed by the Growth and Stability Pact requirements. The latter helps to assess the appropriateness of the fiscal policy and its contribution to the macroeconomic stabilisation, or on the other hand, it unveils its pro-cyclical effects.

The MoFSR has been using the production function approach for the estimation of the potential output and the output gap since 2004. This method was endorsed as the reference method by the European Commission (EC) when convergence programmes are being assessed. However, within the production function approach, it is usually assumed that the growth rate of the technological progress changes smoothly, which is hardly an acceptable assumption for countries with intensive structural changes like Slovakia. In contrast, significant foreign direct investment(FDI) inflow introduced a high volatility into the trend total factor productivity (TFP) in Slovakia. Consequently, the standardized production function method may give misleading and biased results. Therefore, the MoFSR has used experts' adjustments to the trend TFP based on the estimated impact of the FDI on the TFP growth path and partly abandoned the benchmark production function method as recommended by the EC.

As the conventional production function method without the experts' adjustments to the trend TFP has produced counterintuitive results for period of 2006-2008, the MoFSR has decided to work on a complementary model for the identification of the potential output and the output gap. The preference was given to the multivariate dynamic model with unobserved components estimated through Kalman filter (MV Kalman filter). This method is able to overcome several shortcomings of the production function method. It enables to incorporate relationships from economic theory such as the vertical Philips curve and Okun's law. Consequently, the framework is able to model the product and labour market interactions simultaneously. As the model structure is relatively flexible, we add cumulative FDI variable to the model to capture shocks to the trend TFP. Hence, the model "let the data speak" compared to the experts' adjustments in the production function estimates. Our results suggest that ad-hoc adjustments to the TFP estimation in the production function are appropriate in the case of Slovakia. Within our model we also explicitly estimate the non-accelerating inflation rate of unemployment (NAIRU), which is commonly used as an approximation for equilibrium unemployment rate that is the unemployment rate at which inflation remains constant. The problem of high long-term unemployment is perceived as a key curbing factor of the potential growth rate in Slovakia. Our results imply that the NAIRU has fallen significantly since 2001, but still remains at high levels compared with other European economies.

Finally, current economic and financial crisis has reopened research discussion regarding the impact of the deep and long-lasting downturns on the potential output. The consensus in the literature suggests that such downturns result in a permanent loss of its level, while the empirical evidence concerning the effect on its growth rate is ambiguous. In this paper, we identify the size of the cumulative loss of the potential output and the pace of the recovery of the growth rate to its equilibrium rate. The model predicts lower equilibrium growth rates on the forecasting horizon compared with those reached in pre-crisis years.

1. Introduction

Estimating of the potential output and the output gap has been in the centre of research interest for a long time since the proper identification of these unobservable economic indicators is an inevitable input for the formation of the prudent fiscal and monetary policies. In the very broad definition, potential output is the size of the product derived from the supply-side of the economy, which does not drive imbalances – both in the form of emerging price pressures and an external imbalance measured by the current account deficit. Output gap is defined as the relative deviation of the actual output from its potential level. Hence, positive output gap implies an overheating related to the excess of demand on the goods market. On the other hand, the negative output gap identifies a presence of the idle resources in the economy provided that prices are rigid in the short-run. Imbalance on the goods market is directly linked to the excess or slack on the labour market. When output gap is positive, unemployment rate falls below the NAIRU and vice-versa. For more detailed and theoretical discussion on the concepts of the potential output and the output gap see Chagny and Döpke (2001). A brief summary of the economic theory related to the natural rate hypothesis and the NAIRU can be found in Nemec and Vasicek (2007).

Existing literature comprises of four broad groups of the quantitative methods for estimating the potential output and the output gap, for surveys see for instance Dupasquier et al. (1999) and Mc Morrow and Röger(2001). A schematic overview of the existing methods is given in Appendix A. First group, statistical methods consists of an application of statistical filters to the time series of output. The most popular are the Hodrick-Prescott filter (HP) pioneered by Hodrick and Prescott(1997), Band-Pass filter developed by Baxter and King(1995) and the univariate Beveridge-Nelson decomposition introduced by Beveridge and Nelson (1981). These methods have a strong advantage in their simplicity and no demands on data, but are based on purely statistical assumptions about the deterministic or stochastic development of the output time series. Therefore, their direct application may be problematic, since results may lack meaningful economic interpretation. Particularly in case of converging countries statistical filters may misidentify boom and bust periods, as they are not able to distinct between supply and demand shocks. Double-sided filters also face so called end-point problem, which is also a major drawback of popular HP filter. That necessitates the use of the short-term forecasts in the estimation process, as the real-time estimates of the output gap are in focus of the policymakers. To cope with these pitfalls, an interest has been gradually shifted to production function based estimates, the second group of methods.

Unlike the statistical methods, the production function approach has strong theoretical foundations in the economic literature. It relates output to the production factors – labour and capital. Consequently, it also enables to identify determinants of the potential growth - capital accumulation, labour input and the technological augmenting processes, which is undisputedly valuable information for the creation of macroeconomic policies. However, when adopting production function approach, it is still necessary to make several theoretical assumptions. The method requires the choice of the functional form, returns to scale and the representative utilization of production factors. In the majority of empirical studies, the Cobb-Douglas production form is used (Giorno et al. (1995), Morrow and Röger(2001)), which is a special case of constant-elasticity production function (CES) with the unitary elasticity between the production factors. Its popularity comes from its simple log-linear form, in which labour and capital augmenting processes are merged to one variable, TFP. Although the use of the Cobb-Douglas production function is justified only when restrictive assumptions of Kaldor facts hold[1], it has been shown in the empirical literature that it may be a reasonable compromise. In addition, results from a CES function with an elasticity between the factors of 0.8 to 1.2 does not differ significantly from those by Cobb-Douglas technology process. However, the application of the production function approach still has several disadvantages. First, it is subject to data problems, most notably the measure of capital stock. This problem is apparent in case of post-Communist countries, where data from early transition years are relatively unreliable. Second, the need for the use of simple statistical methods - e.g. HP filter is not fully eliminated. Usually, statistical filters are used to obtain the trend TFP and the trend participation rate. Having said that, drawbacks related to the time series filtering are only passed on other economic variables. This issue is discussed in Cerra and Saxena (2000). A discussion of pros and cons of the first two groups of methods is presented in Giorno et al. (1995).

The third group encompasses so called semistructural (or hybrid) methods, which brings together advantages of both statistical and economic-theory based methods. This group includes an extended version of the HP filter – multivariate HP filter, applied for instance by Butler(1996), Conway and Hunt(1997). Commonly, this filter determines the trend and cycle components with additional information from residuals from structural relationships such as the Philips Curve or the Okun's law. A proper calibration of weighting parameters in the minimizing function remains crucial for the estimation. Different weighting schemes were proposed in the above mentioned papers. Structural VARs that were introduced by Blanchard and Quah (1989), model the potential output and the output gap based on the assumption that supply shocks have permanent impact on the production, while demand-side shocks produce only transitory effects. Multivariate Beveridge-Nelson decomposition (Barrel and Sefton(1995), Dupasquier et al.(1999)) models the output gap assuming a co-movement of economic series. Alternatively, a co-movement of output among countries or regions determines the equilibrium relation. Last but not least, the fourth category within semistructural methods are multivariate unobserved components models estimated by MV Kalman filtering technique. These has been most popular recently in the empirical research (Cerra and Saxena (2000), Benes and N'Diaye (2004)). Models have relatively flexible structure and allows for the simultaneous modelling of labour and goods' markets.
Our choice for the MV Kalman modelling framework was motivated by an option of the introduction of country-specific equation for the dynamics of the potential output growth rate. In more detail, we include FDI inflow variable as a determinant of the trend component

[1] Kaldor facts include following assumptions:

1. Per capita output grows at a rate that is roughly constant.
2. The capital-output ratio is roughly constant.
3. The real rate of returns is roughly constant.
4. The shares of labour and capital in national income are roughly constant (over time).

Apparently, for the countries in the convergence process assumptions 1 and 4 are unlikely to be justified.

of the output as the accumulation of FDI was a major reason behind the potential output volatility in the last decade. This way the model enables to overcome previously mentioned pitfalls of the conventional production function method. However, the use of the method still leaves several caveats. Its relatively high flexibility in the assumptions increases demands on the iterative algorithm and its convergence. Besides, having short time series with several structural breaks increases the sensitivity on starting parameters of the filter. There are two strategies how to cope with these difficulties. First, it is possible to use simple OLS estimates for starting values of some parameters (Bencik(2008)). Second, it is meaningful to work with calibrated parameters at least for a subset of parameters, when estimated results cannot be interpreted or are shown insignificant (Benes and N'Diaye (2004)).

Direct measurement of the potential output and the output gap based on available real data presents the last group of methods. Most frequently, data on capacity utilization from Business Surveys (soft data) are used for this purpose. Chagny and Döpke (2001) apply this methodology to Eurozone data.

Empirical research on the estimation of Slovakia's potential output and the output gap includes several papers. The MoFSR compares the HP filter, the production function approach, SVAR estimates and direct measurement from economic indicators in Galabova et al. (2005). The study concludes that the MoFSR officially sticks to the conventional production function approach as the official methodology, being consistent with the EC reference method. NBS also regularly publishes its output gap estimate as a part of its quarterly prognosis. The output gap is derived within the QPM model (see Gavura and Relovsky (2005)), which is primarily aimed at the medium-term prognosis. The comparison of the estimates including various statistical filters, the production function estimates and semistructural methods were presented in Steklacova(2003) and Bencik(2008). Toth(2003) uses signalling approach from real data to obtain the proxy for the output gap. MV Kalman filter estimates can be found in Antonicova and Hucek(2005) and Konuki(2008).

However, very little research has been done to estimate Slovakia's NAIRU. Most simplistic versions of the estimates are in Steklacova(2003). Econometric structural model and the multivariate unobserved components model are presented in Gylanik and Hucek(2009). Galabova et al.(2005) argues that the NAIRU estimates by the MV Kalman filter produced unsatisfactory results, so the MoFSR prefers direct extraction of the potential employment for a purpose of the production function projections.

The rest of this paper is organized as follows. The proposed dynamic model, which is estimated by the MV Kalman filter is introduced in Section 2. Empirical results are summarised in Section 3, including also the estimates from two submodels. Results are then confronted with the MoFSR production function estimates with the focus on the discussion of the economic plausibility of the results and their fit to other economic indicators. Separate subsection is dedicated to the evaluation of the NAIRU estimate. The projected cumulative loss on the potential output induced by the shocks related to the current economic and financial crisis is presented in Section 4. The simulation of alternative scenarios of future development follows. Section 5 concludes and makes the implications for the reference method of the MoFSR.

2. The Multivariate Model

The existing literature is rich in the applications of the MV Kalman filter for the estimation of the cyclical position of the economy. In general, two subgroups of dynamic models can be identified. First, as proposed by famous contribution by Gerlach and Smets (1997), system of equations includes the Philips curve as the signalling equation, which enables to fit the output gap estimate to observed price pressures (or alternatively wage pressures). In the second category of models, information from labour market is extracted through the traditional Okun's law, which is again added to the system as a signalling relationship. Such model was for instance proposed by Cerra and Saxena (2000). Most of the later contributions combine the above mentioned approaches, including two signalling equations. Our model follows this path, being inspired also by applications on the Czech Republic in Benes and N'Diaye (2004), Nemec and Vasicek (2007). The system of the MV filter contains eight equations as follows:

(1) $y_t = \bar{y}_t + ygap_t$

(2) $\bar{y}_t = \bar{y}_{t-1} + \mu_{t-1} + \varepsilon_{\bar{y}}$

(3) $\mu_t = \beta\mu_{t-1} + (1-\beta)(\gamma_1 + \gamma_2 \ln(0.3 + FDICUMpc) + \varepsilon_\mu$

(4) $ygap_t = \phi_1 ygap_{t-1} + \phi_2 ygap_{t-2} + \varepsilon_{ygap}$

(5) $u_t = \bar{u}_t + ugap_t$

(6) $\bar{u}_t = \bar{u}_{t-1} + \varepsilon_{\bar{u}}$

(7) $ugap_t = \alpha_1 ygap_t + \alpha_2 ugap_{t-1} + \varepsilon_{ugap}$

(8) $\pi_t = \lambda_1\pi_{t-1} + \lambda_2\pi_{t+1} + \lambda_3\pi_t^{CPI} + (1-\lambda_1-\lambda_2-\lambda_3)\pi_t^f + \lambda_4 ugap_{t-2} + \varepsilon_\pi$

An overview of model's parameters and their description is given in table in Appendix B. Naturally, having two signalling equations in the model brings the benefit of having more structural information about the economy, but it also may complicates the convergence of the Kalman iterative algorithm. Therefore, we also double-check our results from the model with two sub-models. Sub-model 1 omits the Philips curve, equation (8) from the system. Sub-model 2 excludes the information from the labour market, so it does not include equations (5)-(7) from the system above. In the following, we explain the construction of the individual equations and highlight the difference compared to the standard applications that appear in the literature.

Equation (1) represents a simple identity, which defines that the output is a sum of two unobserved components, trend potential output that is the potential output $\bar{y}$ and the cycle component, the output gap *ygap*[2]. Output is presented by the natural logarithm of the seasonally adjusted real GDP in constant prices adjusted for cigarettes' stockpiling effect as explained in Stability Programme for 2008-2012[3]

The stochastic behaviour of the trend component $\bar{y}$ is defined in the equation (2), which is

[2] Lower case characters stands for the logarithmic transformation elsewhere in this paper.

[3] The MoFSR calculates the total impact of cigarettes' stockpiling in *BOX 3* of Stability Programme for 2008-2012(2009). A reason for this adjustment is that for the sake of a more precise calculation of the output gap , the GDP series has to be net of one-off effects caused by the shift of a part of the excise tax between years, which distorted the GDP growth.

assumed to follow a random walk (RW) with a time-varying drift μ. Alternatively, the potential output may be modelled as the RW with the simple deterministic drift, but we consider a time-varying drift framework more suitable for the description of Slovak data as the convergence process has resulted in the variation of the potential growth rate.

While the equations (1) and (2) are standard in the literature, we modify equation (3) to capture the specific features of Slovak economy. Equation (3) describes that the time-varying drift μ obeys the autoregressive process of order one (AR(1)), reverting to the estimated time-varying equilibrium growth[4]. In order words, the model allows short-run deviations from the equilibrium growth. In the literature, it is assumed that it is a constant, which appears unsuitable for converging economies as the technology progresses do not change smoothly in such countries. Therefore, we prefer to capture this feature of the convergence by time-varying equilibrium growth rate. The latter is defined by the cumulative greenfield FDI[5] as a percentage of GDP and the constant. The proposed transformation of the variable is aimed to satisfy the nature in which FDI contributes to the changes in the potential growth rate as introduced by Chudik and Toth (2004)[6]. Such definition of the term of the equilibrium growth was found significant by the model. As to the parameter β, the higher its value, the more persistent is impact of shocks on the potential growth rate. The alternative specification of this equation present in the literature assumes that the drift term μ follows a simple RW process. However, model omitting the reverting term led to unsatisfactory results.

Equation (4) says that the output gap *ygap* is assumed to be AR(2) process[7], which is common in the literature. Alternatively, it may be tested whether output gaps of trading partners are found significant in the equation, as presented in Konuki(2008). However, this approach assumes an advanced degree of business cycle synchronisation, particularly of demand shocks, which has been doubted by several studies for earlier years of transition (e.g. Fidrmuc and Hagara(2004)). A second option is to include additional gap terms that captures the effects of the real monetary conditions on the output gap (Benes and N'Diaye (2004), Gylanik and Hucek(2008)). Such term combines an influence of a deviation of real interest rate from its equilibrium trend level and a deviation of the real exchange rate from its trend level. Since this methodology introduces additional problems of the measurement of the equilibrium exchange and interest rates, which is more of a scope of the monetary policy, we do not include these variables into our modelling framework.

Equation (5) is once again identity, defining that the seasonally-adjusted unemployment rate u (as measured by the Labour Force Survey) is split into the trend component, the NAIRU $\bar{u}$, and the cyclical component *ugap*. It is worth to note here that the unemployment rate as measured by the Labour Force Survey is based on the national concept, so it does not correspond to the ESA employment concept, which is consistent with national accounts. Still, we consider it being the most valuable indicator of an imbalance on the labour market.

---

[4] Although the notion of time-varying equilibrium growth sounds theoretically wrong, we maintain this name of the variable to keep the standard terminology of the multivariate models intact.
[5] Headline net FDI inflow is adjusted for the privatisation proceeds, which bloated FDI figures massively until 2006. Since this kind of the FDI is not believed to change the growth rate of the potential output, we net them off to obtain so called greenfield investments.
[6] This transformation was chosen subjectively. The intuition it follows says that doubling low FDI stock from yearly years of transition does not imply doubling of the production capacities. On the other hand, the impact from an increase from 0% to 20% should be bigger than the one from 20% to 40% increase.
[7] Hence, parameters satisfy $\varphi_1+\varphi_2<1$.

The NAIRU $\bar{u}$ is modelled as the RW stochastic process in equation (6), which is again standard in the literature. Next equation (7) specifies that the unemployment gap *ugap* is directly linked to the output gap, with parameter α_0 representing the elasticity as in the Okun's law. The persistence of the unemployment gap is captured by the lagged dependent variable and parameter α_1.

The Philips curve pioneered by Philips(1958) is represented by equation (8). In general, there are two ways for capturing supply shocks in the inflation and unemployment relationship. The first strategy is to use an explicit set of determinants of these shocks such as oil price, energy prices etc. and include them in the equation as separate exogenous variables. The second strategy is based on a removal of such influences and to use "narrower" definition of inflation for modelling. We opt for the second strategy and model net inflation excluding fuels and imputed rents in the Philips curve in our system of equations. In more detail, we exclude regulated prices, which significantly distorted CPI in 1990s. We also remove food and fuel prices as these are more driven by exogenous factors rather than macroeconomic policies. Last, imputed rents are also excluded from the modelling as this basket item is consider being an investment rather than consumption. Hence, we prefer net inflation ex-fuels and imputed rents being the best proxy for demand-pulled inflation, which may be linked to the excess or slack of demand on the product (and labour) market. Inflation is measured as the quarter-on-quarter percentage change in annualised terms.

We extend the traditional Philips curve by the forward-looking element inline with the New Keynesian theory derived for instance in Christiano et al.(2001) or Gali and Gertler(1999). Omitting forward-looking expectations π_{t+1} from the equation may lead to downward-biased estimates as economic agents would build their inflation expectations only on previous (higher) inflation as we have witnessed a continuous disinflation trend in the modelled time period (Hurnik and Navratil (2005)). One possibility for modelling forward-looking expectations is to use inflation expectations surveys. Since household surveys have unsatisfactorily short history in Slovakia, the only choice is to use expectations of firms in industry, construction and retail sectors over future selling prices (soft data from Business Surveys). However, such variable was found insignificant in our model. Second method is to assume the perfect foresight of agents, which implicitly means that agents also foresee shocks in the economy. This is obviously very simplistic method of including expectations in the equation, but more sophisticated formulation of expectations goes beyond the scope of this paper. As to other nominal determinants of inflation, there is a direct price channel from imported goods in a small open economy. This is captured by an inclusion of the imported inflation π^f measured by the weighted average of PPI of 14 biggest trading partners of Slovakia[8]. The imported inflation is then adjusted for the development of the corresponding nominal effective exchange rate. Going further, we chose regulated prices inflation π^{CPI} , as a variable, which should identify secondary-round effects in demand-led inflation. In other words, it is a proxy for autonomous inflation. The sum of the nominal inflation determinants – backward and forward-looking expectations, the imported and the autonomous inflation is imposed to be one, inline with the homogeneity condition that secures the existence of the vertical Philips curve in the long-run.

Finally, real determinants of inflation are given by the two-quarter lagged unemployment

[8] Weighted imported inflation index was constructed using PPI indices of the following countries: Austria, Belgium, the Czech Republic, France, Hungary, Germany, Italy, Japan, the Netherlands, Poland, Russia, Switzerland, UK and US. A drawback of this approach is that the price indices include fuel and energy prices, which should be preferably excluded from the variable.

gap[9]. It is also possible to extend the equation by another real variable, the real exchange rate gap, but our model denied the variant with this variable. Alternatively to our methodology, it may be considered to use Elmeskov's (1993) concept, which links together unit labour costs and the real determinant (the output gap or the unemployment gap). This approach was applied to Slovak data by Konuki(2008).

Errors in all equations ($\varepsilon_y, \varepsilon_\mu, \varepsilon_{ygap}, \varepsilon_{\bar{u}}, \varepsilon_{ugap}$ and ε_π) are assumed to be identically, independently normally distributed and uncorrelated, following the standard assumptions in the state space methodology. Models are analytically rewritten into the state space framework and then solved by the Kalman filter. For theoretical details see Appendix C. Model is estimated in Eviews 6 on quarterly data since Q1-1996 until Q4-2009. All variables are seasonally adjusted in Demetra software by X-12 procedures, where applicable. GDP series is adjusted for the cigarettes' stockpiling effect as mentioned above.

3. Empirical Estimates of the Potential Output and the NAIRU

3.1 Estimates of the Potential Output and the Output Gap

The estimated parameters are summarised in Figure 1. Table includes results from the benchmark model and two sub-models as described in previous section.

Fig 1 Values of parameters

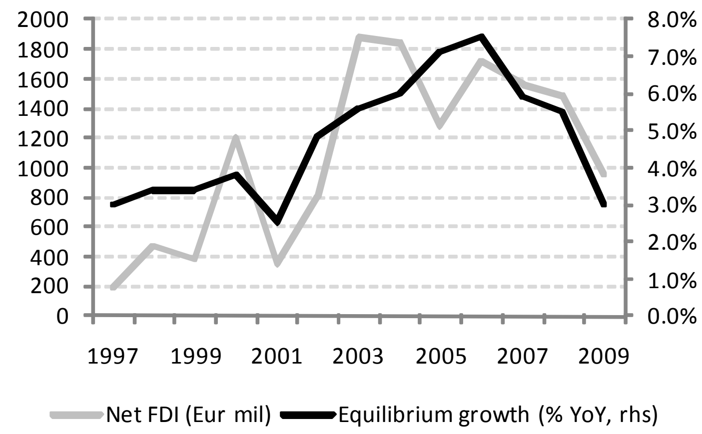
Parameter	Benchmark model	Sub-model 1	Sub-model 2
β	0.699	0.764	0.610
γ_1	0.026	0.027	0.026
γ_2	0.027	0.025	0.026
φ_1	0.688	0.716	1.104
φ_2	0.185	0.131	-0.451
α_1	--0.185	-0.232	
α_2	0.851	0.781	
λ_1	0.518		0.504
λ_2	0.430		0.466
λ_3	0.039		0.028
λ_4 (output gap)			0.319
λ_4 (unemployment gap)	-0.952		

Source: Author's calculation

The value of parameter β should be larger than zero but smaller than one. The higher the value of β, the more persistent is the impact of shocks on the growth rate of the potential output. The estimated value reaches 0.699 in the benchmark model, implying relatively high persistence. Results are similar to those by Konuki (2008), who estimated the parameter at 0.738. Estimates of γ_1 and γ_2 determine how the estimated time-varying equilibrium growth develops, given by the FDI inflow impact on the technology changes in the economy. Implied estimates of the time-varying equilibrium growth rate are plotted in Figure 2.

9 In Sub-model 2, unemployment gap is replaced by the output gap.

Fig 2 Estimated time-varying equilibrium growth (%, YoY)



	Net FDI*) (EURm)	Time-varying equilibrium growth rate (%, YoY)	Potential output (%, YoY)
1998	470.8	3.4	5.8
1999	376.5	3.4	5.7
2000	1201.1	3.8	0.9
2001	356.6	2.5	2.4
2002	791.3	4.8	3.2
2003	1881.0	5.6	4.5
2004	1835.0	6.0	6.2
2005	1280.9	7.1	5.1
2006	1710.9	7.5	7.0
2007	1554.4	5.9	10.1
2008	1487.4	5.5	6.0
2009e	946.2	3.0	1.9

**) Net FDI is adjusted for the privatisation flows.* *Source: The author's calculation*

The equilibrium growth hovered at 3.4% YoY in late 1990s and then gradually accelerated to a peak of 7.5% YoY in 2006. Global slowdown in investments caused a collapse back to an estimated 3% YoY in 2009, a rate comparable to early years of transition. Implied average equilibrium growth for the modelling period 1996-2009 stands at 4.7% YoY, which fits our intuition and also corresponds to the estimates from models with standardised fixed steady-state growth - Konuki (2008) estimated it at 4.4% YoY and Bencik(2008) calibrated it at 4.7% YoY. According to our benchmark model, potential output growth hovered similarly, at 4.8% YoY on average in 1996-2009.

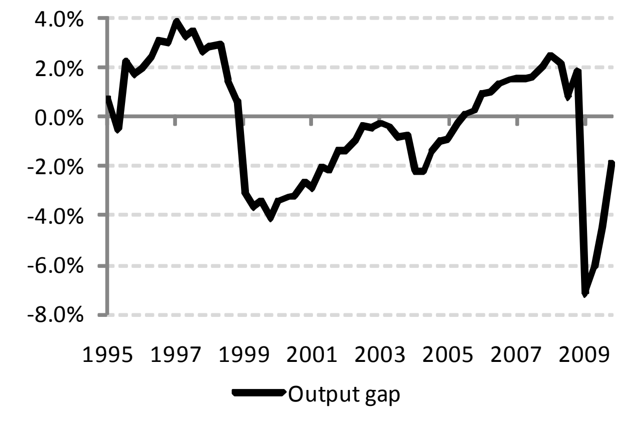
Parameter α_2 estimated in the equation (7) suggests a high persistence of the unemployment gap. The unemployment gap elasticity to the output gap (α_1) has an expected sign and reaches 20%, which looks reasonable. Results within the Philips curve equation appear much more surprising. A derived proportion of backward- and forward looking agents ($\lambda_1/\lambda_1+\lambda_2$ and $\lambda_2/\lambda_1+\lambda_2$ respectively) reach 55:45. The share of forward-looking agents seem unexpectedly high, but is in line with the one estimated by Hurnik and Navratil(2005) for the Czech Republic. The estimated coefficients λ_1 , λ_2 and λ_3 imply surprisingly low coefficient for the imported inflation at 0.013. This looks amazing for such open economy like Slovakia, but it may be explained by the fact that the imported inflation works also through expectations' terms. This reasoning is applicable for the coefficient on the autonomous inflation λ_3 at 0.04 as well, as it is captured by the expectations' terms included in the model. This explains the differences compared to the estimates from different methodologies such as error correction models in Kisidaj and Mihalenko(2006) or Chudik and Toth (2002). Parameter λ_4 measures the elasticity of inflation to the unemployment gap. It has again the expected sign, but its value seems surprisingly high at -0.95. Most of the literature gives an elasticity from -0.4 to -0.6, but our results may be affected by the very narrow definition of the inflation in our model.

The estimated output gap from the benchmark model is reported in Figure 3. The model identifies two economic cycles with two boom and bust periods. First period of an overheating is projected in the second half 1990s, peaking in 1997. Expansionary fiscal policy dominated this period, which resulted in a problem of deepened twin deficits, while economic imbalance was not mirrored in price pressures in that period. This can be reconciled with the fact that an imbalance in a small open economy is typically mirrored in the external accounts. A need for the austerity measures, a beginning of the fiscal consolidation and price deregulation started an era of negative output gap in 1999, which closed only very slowly by 2005. This era was accompanied by the consumption slowdown, a slump in real wages and significant enhancement of the current account deficit. An improvement of business environment, restructuring in banking sector and structural reforms (tax, social and labour market reforms) attracted new greenfield FDIs, which boosted the potential growth of the economy as of 2004. Hence, the recovery was driven by the supply-side impetus, which resulted in an enlargement of the production capacities. The second period of an overheating started in 2006 according to our model and the peak of the boom was achieved in 2008. However, its amplitude was smaller by almost half compared to the one experienced in the second half of 1990s. This supports our view that that the major part of the rapid economic growth in 2006-2008 was driven by changes in the supply-side of the economy, being supported by the development of the observed economic indicators. Both wage and price pressures remained well contained in this period and external accounts were improving as well.

A sharp dip of the output gap is projected in 2009, as a consequence of global demand shock. Theoretical discussions suggest that the long-lasting downturns do also have a negative impact on the potential output and its growth as credit links are frozen, which results in a slow down of investments' formation. That has a negative impact on both the capital accumulation and the productivity growth. This effect has been evident through a significant FDI inflow decrease in 2009. Nevertheless, an estimated slow down of the potential growth rate to 1.9% YoY still implies a deepest slack in the economy of -4.9% in the modelled period. Detailed discussion concerning the impact of the current economic and financial crisis on the potential output and its growth rate follows in Section 4.

Results from alternative Sub-model 1 and Sub-model 2 are summarised in Figure 4 and Figure 5 respectively. The estimated phases of the economic cycles are virtually the same as in the benchmark model. Only two major differences may recognised here. First, Sub-model 1 projects a negative output gap of a smaller amplitude in 1999-2001. Second, the output gap derived from the Sub-model 2 exhibits similar magnitude of the overheating for both 1995-1998 and 2006-2008 boom periods. In this case, we consider the outcome of the benchmark model more realistic, as economic imbalances present in the second half of 1990s were more significant. As to the crisis year 2009, alternative models provide similar results compared to the benchmark model. Models estimate a deceleration of the potential output growth to 1.6% YoY and 2% YoY respectively, implying the negative output gap of -4.5% and -5.4%.

Fig 3 Benchmark model - Output gap (% of potential GDP)

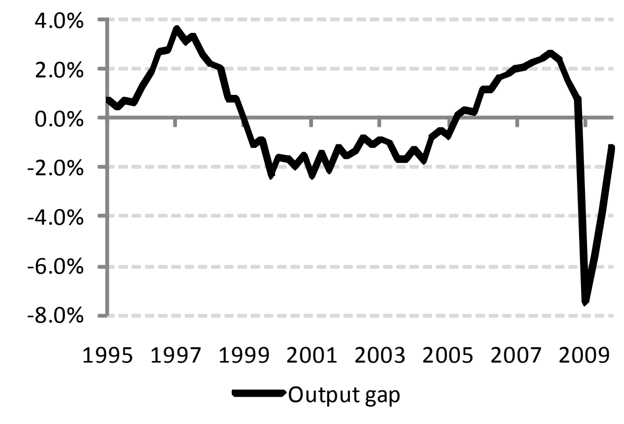


	GDP*) (real growth, %)	Potential GDP (growth, %)	Output gap (% pot. GDP)
2000	1.4	0.9	-3.2
2001	3.5	2.4	-2.1
2002	4.6	3.2	-0.8
2003	4.8	4.5	-0.5
2004	5.0	6.2	-1.7
2005	6.4	5.1	-0.2
2006	9.1	7.0	1.2
2007	9.7	10.1	1.7
2008	7.0	6.0	1.8
2009e	-4.7	1.9	-4.9

**) GDP growth adjusted for cigarettes' stockpiling*

Source: Author's calculations

Fig 4 Sub-model 1 - Output gap (% of potential GDP)

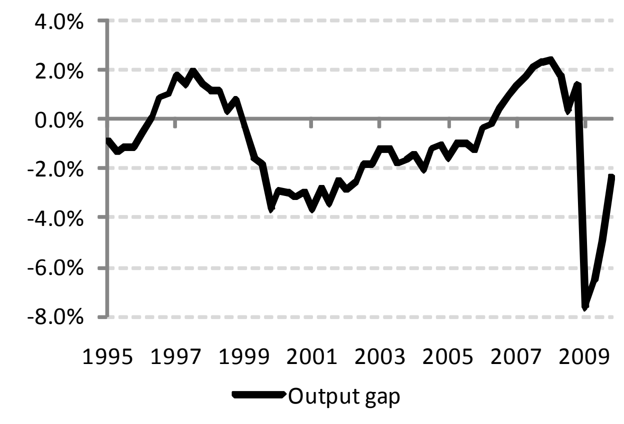


	GDP**) (real growth, %)	Potential GDP (growth, %)	Output gap (% pot. GDP)
2000	1.4	1.9	-1.7
2001	3.5	3.6	-1.8
2002	4.6	4.0	-1.2
2003	4.8	4.9	-1.3
2004	5.0	4.8	-1.1
2005	6.4	5.2	0.0
2006	9.1	7.5	1.5
2007	9.7	8.8	2.2
2008	7.0	7.4	1.8
2009e	-4.7	1.6	-4.5

**) GDP growth adjusted for cigarettes' stockpiling*

Source: Author's calculations

Fig 5 Sub-model 2 - Output gap (% of potential GDP)



	GDP*) (real growth, %)	Potential GDP (growth, %)	Output gap (% pot. GDP)
2000	1.4	2.6	-3.0
2001	3.5	3.6	-3.1
2002	4.6	3.7	-2.3
2003	4.8	3.9	-1.5
2004	5.0	5.0	-1.4
2005	6.4	6.4	-1.2
2006	9.1	7.0	0.2
2007	9.7	8.7	1.9
2008	7.0	6.6	1.5
2009e	-4.7	2.0	-5.4

**) GDP growth adjusted for cigarettes' stockpiling*

Source: Author's calculations

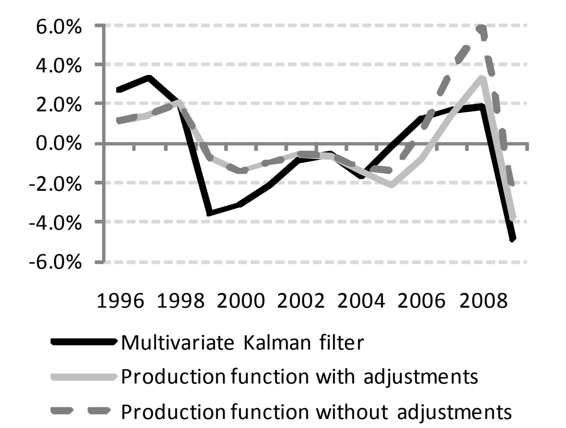
3.2 Comparison to the Production Function Estimates

Here we proceed with the discussion of our results compared to the MoFSR estimates by the production function approach. As the EC's production function estimates are obtained in fully standardised framework without any experts' adjustments to the trend TFP, we also add here a comparison to such estimates.

Figure 6 shows that the projected dynamics of the estimated economic cycle coincides across the methodologies, whereas the magnitude of boom and bust periods differs significantly. First, the MV Kalman filter projects more sleepy economy than the production function in 1999-2001. The bottom of the cycle is estimated at -3.6% in 1999 versus both production function estimates at -1.4%. On the other hand, the MV Kalman filter that the economy was gradually pushed to its potential by 2005, while the production function estimates give a comparable magnitude of idle resources in the economy over the whole course of the downturn until 2006.

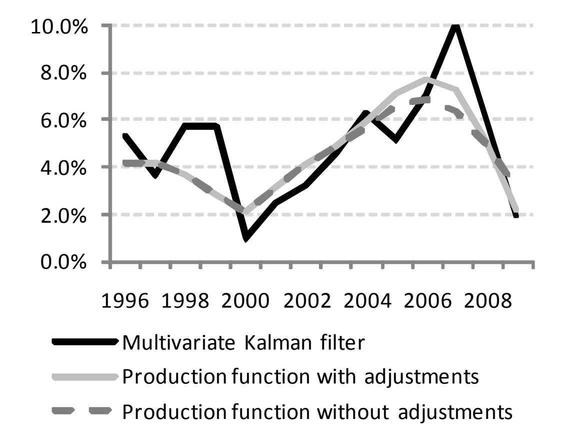
In contrast to the difference in the estimated amplitudes of downturn in 1999-2001, the MV Kalman filter identifies significantly smaller magnitude of an overheating in 2007-2008. Here the major difference arises in the production function estimates without the experts' adjustments to the trend TFP. This methodology estimates a peak of the output gap at a very high 6%, compared with 3.3% given by the production function with the adjustments and 1.8% overheating indicated by the MV Kalman filter[10]. Looking at other economic indicators, such wage and price pressures, a development on the labour market and the improving foreign trade and current account deficits, we assess a small magnitude of the overheating as more likely. Results demonstrate that ad-hoc adjustments to the trend TFP growth are necessary in case of Slovakia, while purely mechanical application of the production function framework may produce counterintuitive results. This is caused by the statistical filtering of the TFP, which cannot capture the variation in the potential output growth rate induced by the ongoing structural changes in the economy.

Fig 6 Output gap (% of potential GDP), comparison to the production function estimates



Source: Author's calculation

Fig 7 Potential output (real growth % YoY), comparison to the production function estimates



Source: Author's calculation

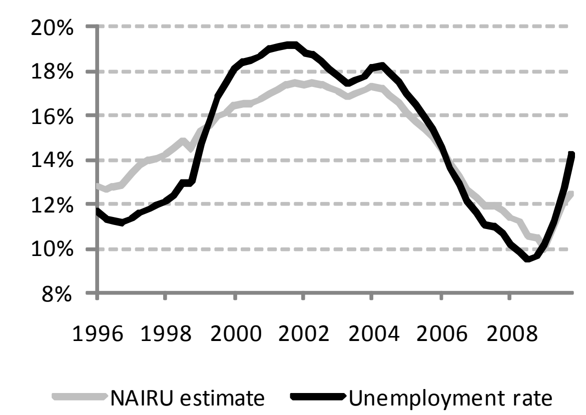
[10] According to the official EC calculations as published in Autumn-2009 European Economic Forecast, the output gap peaked at a very high 9.2% in 2008 in Slovakia, after reaching 7.5% in 2007.

On top of that, the estimates for 2009 diverge substantially again. The most dramatic collapse of the potential growth rate is calculated by the MV Kalman filter, which suggests a slow down to 1.9% YoY, compared with the production function with adjustments at 2.1% and the conventional production function without adjustments at 3.1% YoY. This in turn means that the calculated output gap stands at -5.4% by the MV Kalman filter method versus -3.8% and -2.3% given by the production function methods. Obviously, as shown by Figure 7, this is caused by the smallest volatility of the potential output growth rate allowed by the standardised production function method. On the other hand, determining the potential growth rate through the FDI accumulation, results in the most volatile potential output growth estimated by the MV Kalman filter. Overall, it is obvious that such substantial differences would lead to noticeably different evaluation of the policy stances and advises for the policy formulation.

3.3 The NAIRU Estimates

Looking at the estimates related to the labour market, it is worth to highlight that the results confirm a presence of the hysteresis on the labour market in Slovakia. In order words, the NAIRU estimate follows the actual unemployment rate relatively closely. In other words, deviations of the unemployment rate from the NAIRU imply the same trend of the unemployment and the NAIRU. Theoretically, the hysteresis presence suggests that changes in the aggregate demand have long-term effect on the unemployment rate that is they have also an impact on the NAIRU. This fact and its consistency with the concept of the Philips curve was studied by Ball(1997, 1999) . The estimated NAIRU from our main model and the alternative Sub-model 1 are displayed in Figure 8 and Figure 9, respectively.

Fig 8 Benchmark model – the NAIRU estimate (% of labour force)



	Unemployment rate (% of labour force)	NAIRU (% of labour force)	Unemployment gap (ppt)
2000	18.4	16.6	1.9
2001	19.1	17.2	1.9
2002	18.5	17.4	1.2
2003	17.6	17.0	0.6
2004	17.9	17.0	1.0
2005	16.1	15.5	0.6
2006	13.3	13.5	-0.2
2007	11.1	11.9	-0.9
2008	9.7	10.9	-1.2
2009e	12.0	11.4	0.7

**) Unemployment rate according to Labour Force Survey, seasonally-adjusted* *Source: Author's calculation*

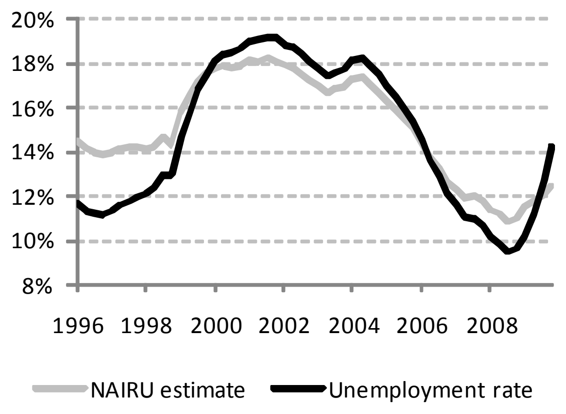
Similarly to the output gap estimates, the modelled interval may be split into four sub-periods. First era with the negative unemployment gap lasted from 1996 to late 1998. In that period, the NAIRU was gradually growing to 15%, while the unemployment rate was squeezed down to roughly 12% by the expansionary policies. A high value of the NAIRU confirms a complicated heritage from communists' era and rigid features of Slovak labour market. As an improvement of the business environment and the restructuring of the companies took place only later, there was basically no helpful impetus for the structural unemployment to fall down. In turn, employment was maintained unsustainably high, together with the postponed price deregulation, which complicates the investigation of inflation – unemployment relationship in this period. We would suggest here that NAIRU was in reality even higher at around 16-17% than the results suggested by the benchmark model.

According to our estimates unemployment rate and the NAIRU started growing rapidly in late 1998, when the economy experienced a negative demand shock. A hysteretic feature of the labour market resulted in an increase of the long-term unemployed and the NAIRU peaked at above 17% in 2002, while the unemployment rate topped at around 19% in 2001. A reversal of the negative trend in the NAIRU realised only in 2003. In our opinion, a continuation of positive trend can be attributed to the structural reforms mentioned earlier, which enables a rebound of the private sector, attraction of greenfield FDI and an enlargement of the production capacities. Changes to the Labour Code helped to remove or partly limit several rigidities present on the labour market, which are traditionally believed to induce the hysteresis (presence of the Labour Unions, hiring and firing practices, flexible working schemes, unemployment benefit etc.). Besides, new active labour market policies were introduced by the government as of 2004, which are assumed to diminish mismatches on the labour market and pull in the long-term unemployed back onto the market. Nevertheless, we believe that the demand boost from newly built production capacities was the main reason behind the NAIRU decrease since 2003.

In 2006, the unemployment rate fell again below the NAIRU for the first time since 1998, bottoming at 11% in 2008. A declining trend of the NAIRU was stopped by the global crisis and the NAIRU is estimated to nudge up slightly to 11.4% in 2009. A beginning of the negative trend could confirm the hypothesis that the labour market becomes more rigid during the long-lasting downturns on the back expansionary social policy, which is generally believed to have a larger impact on an increase of the long-term unemployment rate in countries with hysteretic features of the labour market.

Concerning, the NAIRU estimates from Sub-model 1, results are broadly similar to the benchmark model. The only exception is the period of the second half of 1990s, when the model projects higher NAIRU, hovering constantly at 14% in 1996-1998. This supports more our intuition as described above, as the models may potentially underestimate the NAIRU due to artificially biased inflation-unemployment relationship. Elsewhere, the model gives pretty similar turning points as the benchmark model. The NAIRU top is estimated at 18% in 2001, following with a gradual decline to 11% in 2008. Again, the crisis year 2009 resulted in an increase of the NAIRU to 11.9%, while the unemployment gap turned marginally positive.

Fig 9 Sub-model 1 – the NAIRU estimate (% of labour force)



	Unemployment rate (% of labour force)	NAIRU (% of labour force)	Unemployment gap (ppt)
2000	18.4	17.8	0.6
2001	19.1	18.1	1.0
2002	18.5	17.6	0.9
2003	17.6	16.9	0.7
2004	17.9	17.1	0.8
2005	16.1	15.7	0.5
2006	13.3	13.5	-0.2
2007	11.1	12.0	-0.9
2008	9.7	11.1	-1.3
2009e	12.0	11.9	0.1

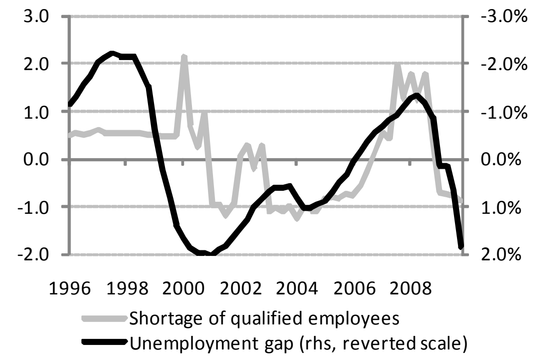
**) Unemployment rate according to Labour Force Survey, seasonally-adjusted* *Source: Author's calculation*

At this point, it is necessary to find arguments supporting our results, which would confirm a presence of significantly time-varying NAIRU. Opponents may suggest that the conceptual definition of the NAIRU speaks in favour of a little variation of this economic indicator and that the implied unemployment gap has surprisingly small magnitude. First, results by Gylanik and Hucek (2009) also acknowledged a close co-movement of the unemployment rate and the NAIRU. Yet, their NAIRU estimate by the MV Kalman filter has a higher standard deviation than those from our models. Second, similarly to their study, we proceed with checking of a reliability and interpretability of our results with the development of other labour market indicators. Below, we selected four indicators which may indicate an emerging imbalance on the labour market.

Figure 10 shows the comparison of the estimated unemployment gap (on a reverted scale) to the soft indicator from Business Surveys data published by SUSR. In the relevant question entrepreneurs are asked "Which factors do curb your business activities?". Among the suggested answers, one of the options is "a shortage of qualified employees on the market". This business survey covers industry, construction, retail and services sectors. However, until 2002 only data for industry and construction are available. In our weighted indicator, we attached weights to the sectors as they are attributed in economic sentiment indicator (ESI). A positive value of indicator means difficulties in hiring qualified labour and vice-versa. As shown by the chart, the estimate of the unemployment gap copies the dynamics of the indicator relatively closely only from 2003. This can be explained by the exclusion of important sectors from the survey indicator until 2002 as written above.

Second analytical comparison is given in Figure 11. The unemployment gap is depicted against the standardised vector of number of vacancies as published by the UPSVaR. Unlike previous conclusion, here we may observe a significant correlation of the series over the whole modelled period. However, the chart suggests that our measure of the unemployment gap lags systematically behind the vacancies indicator by approximately three quarters.

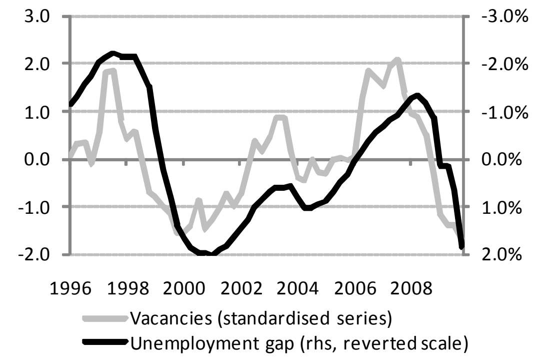
Fig 10 Unemployment gap (%) vs. a shortage of qualified employees*)



**) A shortage of qualified employees is a series published in Business Surveys indicators published monthly by SUSR, representing a soft indicator. This series includes industry, construction, retail and services sectors weighted as in ESI.*

Source: SUSR, Author's calculation

Fig 11 Unemployment gap (%) vs. vacancies (standardised series,))**

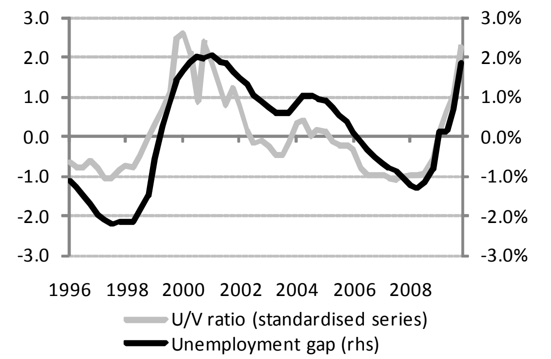


***)Data on vacancies are monitored by the UPSVaR., However, as private companies do not have obligation to report the number of vacancies, level data are not applicable for further analysis.Therefore, we standardise the time series, which in turn implies that zero value does not necessarily means the balance on the market.*

Source: UPSVaR, Author's calculation

In Figure 12, we compare the unemployment gap dynamics against the standardised series of the unemployed to vacancies ratio, which is commonly used for the description of the cyclical position of the labour market. The chart again confirmed that our measure of the unemployment gap properly identifies phases of the economic cycle. Last but not least, we look at the development of the long-term unemployment rate, which may serve as a proxy for the NAIRU dynamics. Figure 11 again gives evidence that our results do find a support in the observed empirical data. It also suggests that the recent NAIRU pick-up is already mirrored in the increased rate of the long-term unemployed.

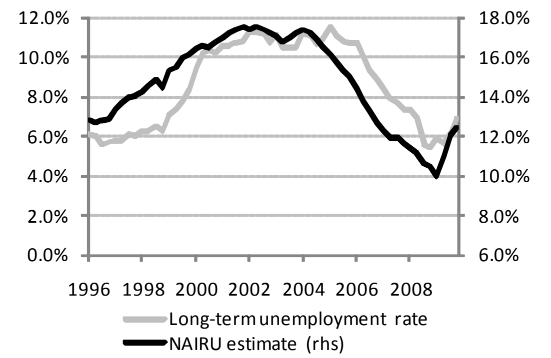
Fig 10 Unemployment gap (%) vs. U/V ratio (standardised series *))



**) Data on vacancies are monitored by the UPSVaR., However, as private companies do not have obligation to report number of vacancies. Therefore, level data are not applicable for further analysis .Our analytical series represents the number of unemployed per one vacancy in the standardised terms. Standardising of the variable implies that zero values do not necessarily represent a balance on the labour market.*

Source: UPSVaR, SUSR, Author's calculation

Fig 11 NAIRU estimate (%) vs. long-term unemployment rate (%)



Long-term unemployment is defined as the unemployment that lasts over one year.

Source: SUSR, Author's calculation

4. Simulation of the Cumulative Loss on the Potential Output Resulting from Current Economic and Financial Crisis

In most of the countries, a sharp decline of GDP is the most pronounced impact of the current financial and economic crisis. In hand with this, also the potential output is negatively affected. However, according to the literature, the impact on the growth rate of the potential output is ambiguous. In general, three variants of the future development are possible:

- After a decrease of the potential output, its growth rate will recover in the medium-term. As a consequence, the loss on the potential output gradually evaporates.
- Potential output records a permanent shortfall, but the impact on the growth rate is only temporary. That said, it will follow with its past growth rates after some time. In this case, an impact of the crisis is measured as the cumulative loss on the potential output, which stabilizes in the long-term.
- The crisis has a permanent negative effect both on the level and the growth rate of the potential output. Hence, the cumulative loss on the potential output does not stabilize even in the long-term.

Although it is very difficult to decide which scenario is most likely in case of Slovakia, we see several arguments why we put a preference to the **second variant**. First, Slovakia has had relatively healthy and stable financial sector, which did not need any public injection in 2009. Second, we also believe that the attracted foreign-owned production capacities, which have settled in the country, are likely to stay here in the long-term. This argument may be supported by the presence of the industrial clusters, geographical proximities to the main exports' markets and attractive business environment (tax system)[11]. Second, Slovakia still has preserved the competitiveness thanks relatively low labour costs and the availability of skilled labour. Last but not least, the Eurozone membership helped to eliminate the exchange rate volatility and an environment of stable and low interest rates.

In the following exercise we simulate the estimated loss on the potential output caused by the current global economic downturn. The size of the negative demand shock is given by the model and its negative impact on the potential output growth rate is identified in 4Q08-4Q09. Baseline "ex-crisis" scenario excludes these identified shocks and both projected scenarios assume no further negative shocks hitting the economy as of 1Q10. As from 2010, our projection simply uses GDP and FDI forecasts from the MoFSR prognosis published in January 2010. As of 2013, only indicative forecasts of both these indicators are used. Former is used only for the purpose of the output gap calculation and the latter enters the simulation of the potential growth rate recovery. Such simulation naturally takes into account that investment formation driven particularly by foreign-owned companies will slow down in the medium-term horizon compared to the pre-crisis years. That way we try to capture widely believed fact that the stock of FDI will be accumulated at a relatively moderate pace as medium-sized investments' projects are more likely to take place in the forecasting period.

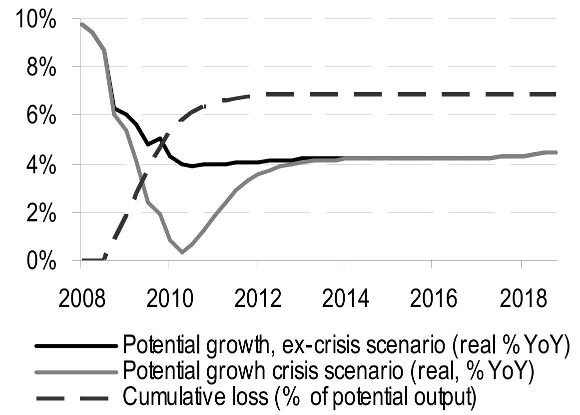
Results are summarized in Figure 12. As shown by the chart, identified shocks will fully evaporate from the potential growth rate only in 2013, which is given by β parameter in the

[11] Slovakia managed to attract some FDI projects even in 2009, e.g. new production line at VW Bratislava, Au Optronics.

model. The potential growth rate is estimated to stabilize at 4.2% YoY as from 2013 after being squeezed down to below 2% YoY in 2009-2010. It is worthy to note here that the estimated average of the potential growth in 1996-2008 reached 5.8% YoY. Alongside with that, the cumulative loss on the potential output will rise to 7%. Naturally, should one calculate the cumulative loss using the long-term pre-crisis estimated average of the potential growth rate, the loss climb to above 10%.

To conclude the simple forecasting exercise, we have to note here that the projected output gap closes significantly slower than the one given by the MoFSR production function estimates. Economy could operate at its potential only in 2017.

Fig 12 Estimated cumulative loss on the potential output (% of the potential output)

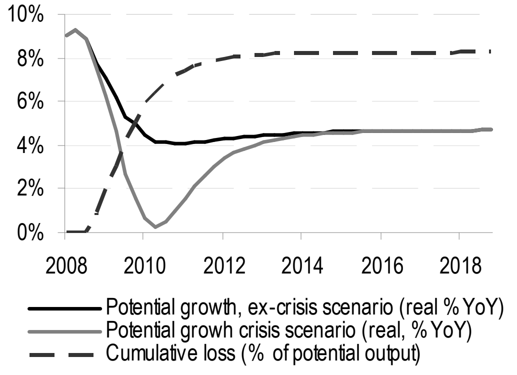


	Potential GDP,ex-crisis scenario*) (real growth, %)	Potential GDP, crisis scenario *) (real growth, %)	Projected Output gap (% pot. GDP)
2008	6.2	6.0	1.8
2009e	5.0	1.9	-4.9
2010F	3.9	1.2	-3.3
2011F	4.1	3.3	-3.2
2012F	4.2	4.0	-2.7
2013F	4.2	4.2	-2.2
2014F	4.2	4.2	-1.9
2015F	4.2	4.2	-1.2
2016F	4.2	4.2	-0.6
2017F	4.2	4.2	0.0

**) GDP growth adjusted for cigarettes stockpiling. Calculations beyond 2013 are based on indicative estimates of GDP growth and FDI and are not a part of the MoFSR official macroeconomic forecast.* *Source: Author's calculations*

Alternative models imply a bit more extreme scenarios. As can be seen from Figure 13, Sub-model 1 indicates a stabilisation of the potential growth rate at the estimated ex-crisis pace only in 2018 (implied by higher estimated value of β parameter). The growth rate is projected to recover to 4.8% YoY, higher than indicated by the benchmark model. On the other hand, the cumulative loss reaches over 8%. In contrast, Sub-model 2 suggests that the growth rate may recover to ex-crisis scenario already in 2013 (as the estimated β is the smallest one). At the same time, this model projects the lowest potential growth rate at 4% YoY as from 2013. Implied loss on the potential output accumulates to 'only' 6.3% (Figure 14).

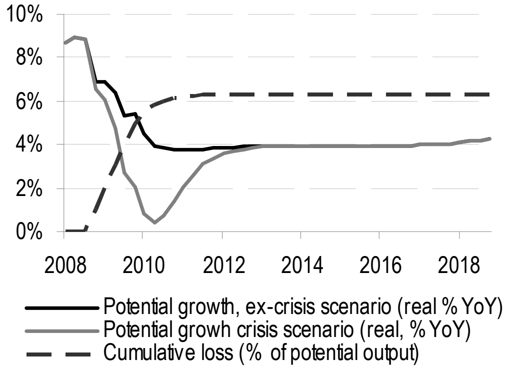
Fig 13 Estimated cumulative loss on the potential output (% of the potential output), Sub-model 1



**) GDP growth adjusted for the cigarettes stockpiling. Calculations beyond 2013 are based on indicative estimates of GDP growth and FDI and are not a part of the MoFSR official macroeconomic forecast.*

Source: MoFSR, Author's calculation

Fig 14 Estimated cumulative loss on the potential output (% of the potential output), Sub-model 2



**) GDP growth adjusted for the cigarettes stockpiling. Calculations beyond 2013 are based on indicative estimates of GDP growth and FDI and are not a part of the MoFSR official macroeconomic forecast.*

Source: MoFSR, Author's calculation

5. Conclusion

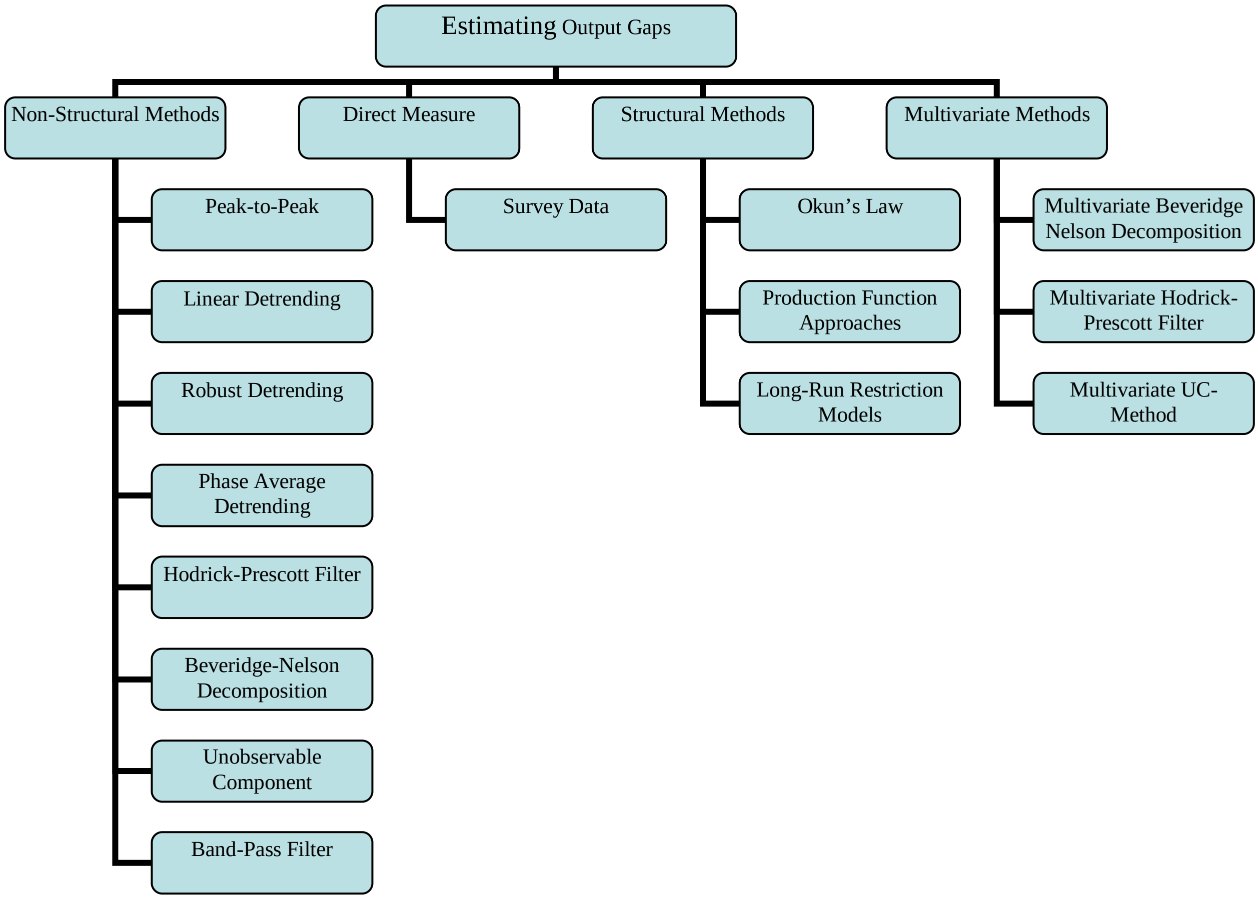
In this study we developed a multivariate model with unobserved components for the purpose of the output gap and the NAIRU estimation. By using the MV Kalman filtering technique, we tried to find a justification for ad-hoc experts' adjustments to the trend TFP in the application of the production function approach in the case of Slovakia. Our results suggest that the MV Kalman filter estimates of the output gap are similar to those by the modified production function, while the conventional production function framework gives significantly different results. This outcome is provided by the fact that the MV Kalman filter modelling framework enables to capture a variation in the potential growth rate in contrast to the conventional production function approach, which assumes a smoothly changing productivity growth. As a consequence, the production function without the experts' adjustments may produce misleading results in case of countries with significant discontinuous supply-side shocks.

As the MoFSR aims to follow the EC's recommendations, it will continue using the production function approach including the manual interventions to the TFP dynamics as the reference method for the output gap calculation. A developed multivariate model gives robust results, which can be well reconciled with other observed economic imbalance indicators. Therefore, it will serve as a complementary tool for justification of ad-hoc adjustments to the trend TFP in the reference production function estimate for the internal purposes of the MoFSR.

In the last chapter of this paper, we simulated the estimated cumulative loss on the potential output which resulted from the current economic and financial crisis. Based on several assumptions, we consider a permanent impact on the level of the potential output and temporary effects on its growth rate as the most likely scenario in the case of Slovakia. Model's simulation supports such scenario. The alternative calculations from the model indicate that the loss could stabilise at 6-8% of the potential output during the upcoming decade. Negative demand shock may fully evaporate from the potential growth rate only in 2013. At the same time, the potential growth rate will accelerate gradually, but will not reach the equilibrium growth rate recorded in pre-crisis years. This is primarily caused by the global slowdown in the investment formation and frozen credit links, which will contribute to a relatively moderate pace of FDI inflow, a major driver of enlargement of production capacities in Slovakia. Finally, our indicative results imply that the economy could operate below its potential over the whole forecasted period.

Appendix A

Methods to estimate output gap.



Appendix B

Description of parameters.

Description of parameters	
Parameter	Description
β	Persistence of shocks to the potential growth rate
γ_1, γ_2	FDI inflow impact on the equilibrium growth rate
φ_1	AR(1) coefficient in the output gap equation
φ_2	AR(2) coefficient in the output gap equation
α_1	Unemployment elasticity to the output gap
α_2	Persistence of the unemployment gap
λ_1	Weight of backward-looking expectations in the Philips curve
Λ_2	Weight of forward-looking expectations in the Philips curve
Λ_3	Weight of autonomous inflation proxied by CPI inflation in the Philips curve Elasticity of inflation to the unemployment gap (in the Benchmark model),
Λ_4	Elasticity of inflation to the unemployment gap in Sub-model 2

Appendix C

The state space specification enables the representation and estimation of various time series models such as the ARIMA models. The use of the state space form to represent dynamic systems allows to model and estimate unobservable variables along observable components. Furthermore, a popular recursive algorithm called the Kalman filter can be used to analyse models in the state space form.

Two type of processes are present in the state space framework. The first follows the measurement equation (1), where the g-vector y_t represents the observed process and the s-vector x_t is the state vector of the system, representing unobservable variables. Furthermore, u-dimensional exogenous vector z_t and an error term v_t are included in the equation.

$$y_t = \beta_t x_t + \gamma_t z_t + v_t \qquad (1)$$

The state variables in vector x_t evolve according to the motion (or state transition) equation (2), which includes L exogenous variables in vector w_t and an error term u_t.

$$x_t = T_t x_{t-1} + \delta_t w_t + G_t u_t \qquad (2)$$

The error terms u_t and v_t are assumed to be independent and normally distributed with zero mean and with covariance matrices R_t and Q_t, so that:

$$\begin{bmatrix} v_t \\ u_t \end{bmatrix} \sim N\left[0, \begin{bmatrix} R_t & 0 \\ 0 & Q_t \end{bmatrix}\right]$$

Additional assumptions are that initial value of x_t is a random variable from normal distribution, independent of u_t and v_t, variables z_t and w_t are exogenous and independent of all u_t and v_t, parameters are assumed to be known and finally that the model fully represents the analysed system.

The Kalman filter uses the measurement and motion equations together with the results of the Bayes' theorem to construct the conditional density function of unobservable variables under the above stated assumptions. In particular, it is a recursive algorithm, which uses new information interfered from the observable variables to update one-step ahead estimates of state mean and variance. It proceeds in two steps. First, an one-step ahead estimate of state variable $\hat{x}_t$ and its covariance matrix $\hat{P}_t$ is calculated based on current observation. Next, estimate of observable variable $\hat{y}_t$ and its covariance matrix $\hat{H}_t$ is obtained.

$$\begin{aligned}
\hat{x}_t &= T_t x_{t-1} + \delta_t w_t \\
\hat{P}_t &= T_t P_{t-1} T_t^T + G_t Q_t G_t^T \\
\hat{y}_t &= \beta_t \hat{x}_t + \gamma_t z_t \\
\hat{H}_t &= \beta_t \hat{P}_t \beta_t^T + R_t
\end{aligned}$$

In the second step these estimates of state variable and its covariance matrix are updated using the realisation of the observable variable in the following period, where k_t is the Kalman Gain.

$$x_t = \hat{x}_t + k_t(y_t - \hat{y}_t)$$
$$k_t = \hat{P}_t \beta_t^T H_t^{-1}$$
$$P_t = \hat{P}_t - k_t \beta_t \hat{P}_t$$

However, for the realisation of the Kalman filter the parameter values need to be known. Similarly, the calculation requires the initial value for the estimate of the state variable and its covariance matrix $\hat{x}_0$ and $\hat{P}_0$ respectively.

References

[1] Antoničová Z., Huček J.(2005), *"Viacrozmerný filter s nemerateľnými stavmi"*, Biatec 6/2005

[2] Ball L. (1997), *"Disinflation and the NAIRU"*, In: Romer Ch. – Romer D., *"Reducing Inflation: Motivation and Strategy"*, University of Chicago Press, 1997, p.167-185

[3] Ball L. (1999), *"Aggregate Demand and Long-Run Unemployment"*, Brookings Papers on Economic Activity, 1999, p.189-251

[4] Barrel R., Sefton J. (1995), *"Output Gaps. Some Evidence from the UK, France and Germany"*, National Institute Economic review, 1995, 65-73

[5] Benčík M. (2008), "Metódy detekcie nerovnováhy v reálnej ekonomike SR", NBS, Working paper 2/2008

[6] Benes J., N' Diaye P. (2004), *"A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic"*, IMF Working Paper, 2004/45

[7] Baxter M., King R. (1995), *"Measuring Business Cycles: Approximate Band-Pass Filters for Time Series"*, NBER Working paper no.5022 /1995

[8] Beveridge S., Nelson C.R. (1981), *"A New Approach to Decomposition of Economic time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle"*, Journal of Monetary Economics 7(2), 151-174

[9] Blanchard O., Quah D. (1989), *"The Dynamic Effects of Aggregate Demand and supply disturbances"*, The American Economic Review, Vol.79 No.4

[10] Butler L. (1996), *"A Semistructural method to Estimate Potential Output: Combining Economic Theory With a Time Series Filter"*, Bank of Canada, The Bank of Canada's New Quarterly Projection Model (QPM), Part 4, 1996

[11] Cerra V., Saxena S.C. (2000), *"Alternative Methods of Estimating Potential Output and the Ouput Gap: An Application to Sweden"*, IMF Working Paper, 2000/59

[12] Chagny O., Döpke J. (2001), *"Measures of the Output Gap in the Eurozone: An Empirical Assessment of Selected Methods"* , Vierteljahrshefte, Wirtshaftsorschung 70. Jahrgang 2001, Heft 3, p. 310–330

[13] Christiano L., Eichenbaum M., Evans C.L. (2001), *"Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy"*, Cleveland FED, WP 2001/107

[14] Chudík A., Tóth J. (2002), *"Slovak Inflation and Wage Models"*, ING Bank Bratislava, Economics paper no.1

[15] Chudík A., Tóth J. (2004), *"Slovak Foreign Trade Model"*, ING Bank Bratislava, Economics paper no.3

[16] Conway P., Hunt B. (1997), *"Estimating Potential Output: A Semi-structural Approach"* , Federal Reserve Bank of New Zealand Discussion Paper no.9, 1997

[17] Denis C., Mc Morrow K., Röger W., *"Production Function Approach to Calculating Potential Growth and Output Gaps - Estimates for the EU Member States and the US"*, Economic Paper no. 176/2002, European Commission

[18] Denis C., Grenouilleau D., Mc Morrow K., Röger W. (2006), *"Calculating potential growth rates and output gaps – A revised production function approach"*, European Economy, EC, Economic Paper no. 247/2006

[19] Dupasquier C., Guay A.,St-Amant P. (1999), *"A Survey of Alternative Methodologies for Estimating Potential Output and the Output Gap"*, Journal of Macroeconomics, 21, 577-595

[20] Elmeskov J. (1993), *"High and Persistent Unemployment: Assessment of the Problem and Its Causes"*, OECD WP 132/1993

[21] Fidrmuc J., Hagara E. (2004), *"Similarity of Supply and Demand Shocks in the European Union and Acceding Countries (implications for the Czech Republic and* Slovakia)", Politiká Ekonomie 2/2004

[22] Galabová M., Kucserová J., Valachy J., Ódor Ľ., Krajčír Z. (2005), *"Odhad produkčnej medzery a štrukturálneho salda verejných financií v SR"*, IFP, MFSR, Ekonomická analýza 3

[23] Gali J., Gertler M. (1999), *"Inflation dynamics: A structural Econometric Analysis"*, Journal of Monetary Economics, 1999, no. 44, 195-222

[24] Gavura M., Reľovský B. (2005), *„Jednoduchý model transmisného mechanizmu ekonomiky SR, jeho štruktúra a vlastnosti"*, QPM model, NBS, Biatec 13/2005

[25] Giorno C., Richardson P., Roseveare D., Noord P. van den (1995), *"Estimating Potential Output, Output Gaps and Structural Budget Balances"*, OECD WP, no. 152.

[26] Gylánik M., Huček J. (2009), *„Odhad NAIRU v slovenskej ekonomike"*, NBS, Monetary Policy Deparment, 10/2009

[27] Hodrick R.J., Prescott E.C. (1997), *„Post War U.S. Business Cycles: An Empirical investigation"*, Journal of Money, Credit and Banking, 29, 1-16

[28] Hurnik J., Navratil D. (2005), *„Labour Market Performance and Macroeconomic Policy: The Time Varying NAIRU in the CR"*, Czech Journal of Economics and Finance, 55, 2005, no.1-2

[29] Kichian M. (1999), *„Measuring Potential Output within a State-Space Framework"*, Bank of Canada Working Paper, 1999/9

[30] Kišidaj P., Mihalenko M. (2006), *„Modelovanie inflácie v podmienkach SR (mark-up model)"*, IFP, MFSR, Ekonomická analýza 10

[31] Konuki T. (2008), *„Estimating Potential Output and the Output Gap in Slovakia"*, IMF, WP/08/275

[32] Mc Morrow K., Röger W. (2001), *“Potential Output: Measurement Methods, “New” Economy Influences and Scenarios for 2001-2010 – A Comparison of the EU15 and the US”*, EC, Economic Paper no. 150/2001

[33] Nemec D., Vasicek O. (2007), *„Alternative Estimates of NAIRU for the Czech Economy: Implications for Economic Growth and Stability“*, Masaryk University Brno, 2007

[34] Philips A.W.(1958), *„The Relation between Unemployment and the Rate of Change of Money Wages Rates in the United Kingdom“*, Economica, November, 283-299

[35] Štekláčová L. (2003), *„Meranie produkčnej medzery v SR“*, FMFI UK, Diploma thesis, 2003

[36] Tóth (2003), *„Slovak Output Gap“*, ING Bank Bratislava, Economics paper no.2