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Forecasting The Term Structure of Interest Rates in Slovakia

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FORECASTING THE TERM STRUCTURE OF INTEREST RATES IN SLOVAKIA

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Abstract

We evaluate forecasts of the term structure of Slovak government bond yields at horizons ranging from one to 36 months in the out-of-sample period from January 2009 through December 2016. We split the forecasting exercise into two steps. First, we forecast the term structure of German government bond yields using a dynamic Nelson-Siegel (DNS) model and a three-factor model of the yield curve with economically motivated factors which we call a "trend–cycle" model. In the second step, we forecast the term structure of spreads to German government bond yields. This "sum-of-the-parts" forecasting model delivers forecasts that outperform the random walk benchmark at most forecasting horizons. A combination of the trend-cycle model for forecasting German government bond yields and an AR(1) model for spreads delivers the best results at horizons up to one year. At longer horizons, the trend-cycle model combined with the random walk spread forecast is the best forecasting model. The main source of the improvement in forecasting performance relative to the random walk benchmark is the negative correlation between credit spread forecast errors and forecast errors on German government bond yields, i.e. negative surprises to the future path of the policy rate tend to coincide with positive surprises to sovereign credit spreads. We show that the forecasting performance of the dynamic Nelson-Siegel model deteriorates significantly in periods when short-term interest rates are at or close to the effective lower bound.

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1 Introduction

Forecasting interest rates is an important exercise not only for investors but also for central bankers, corporations, and governments. Not surprisingly, the importance of forecasting interest rates is only matched by difficulty in finding predictive models that work out-of-sample. This is reflected in the widely accepted conclusion in the empirical literature, namely that most forecasting models do not meaningfully outperform a naive random walk benchmark. The difficulty in reliably forecasting interest rates has likely multiple sources. First, interest rates tend to undergo regime shifts, which are most often driven by long-lasting changes in monetary and fiscal policies. This form of instability is inherently hard to model. Another, more recent, complicating matter has arisen when most of the major central banks set their policy rates to or close to the effective lower bound for an extended period of time. Such an environment has made outperforming the naive benchmark more difficult as the mean-reverting (business cycle) movements in the policy rate have largely been eliminated.

The focus of our analysis is determined by two aspects. First, and most importantly, we are interested in better understanding the behaviour of Slovak government bond yields for the purposes of efficient government debt management. To this end, we want to not only document the degree of predictability of Slovak government bond yields but also understand the source of it. Second, the case of Slovakia is distinctly different from other euro zone members in that it is a converging country with a relatively low level of public debt sharing a common currency. This gives rise to two sources of variation in Slovak government bond yields. First source is related to the monetary policy of the European Central Bank (ECB) and is common to all eurozone member countries. Second source of variation is specific to Slovak economy and mostly reflects fluctuations in the sovereign credit risk and the liquidity premium.

The goal of this paper is to document the predictability of Slovak government bonds and to understand the economic sources of the predictability. To this end, we evaluate the performance of a small number of forecasting models applied to Slovak government bond yield data in the out-of-sample period January 2009 through December 2016. This period, despite being relatively short, spans several monetary policy regimes and two financial crises. Similar to previous literature, we compare forecasting models to random walk. Our focus lies in understanding the predictability at medium and long-term horizons. These facts determine our modelling choices. Instead of running horse races with a large number of forecasting models, we pre-select a small number of forecasting models that help us locate the source of predictability. We first study the predictability of German government bond yields which serve as a benchmark for pricing Slovak government bonds. Given its widespread use, we start with the dynamic Nelson-Siegel model to forecast German government bond yields. Our second model for forecasting German yields decomposes the term structure into economically interpretable quantities:

long-run inflation expectations, ex-ante real rate variation and a term premium factor. This decomposition allows us to forecast each quantity separately and also understand the source of predictability. In the next step, we evaluate the predictability of the spreads of Slovak bonds relative to their German counterparts. We use two models to forecast the term structure of sovereign credit spreads. First, we apply a simple autoregressive model with one lag for each maturity separately. Second, we add two financial variables (the VIX index and the slope of the German yield curve) to the autoregressive component.

We document that a combination of the restricted three factor model with any of the forecasting models for the credit spread (including the random walk) delivers better forecasts of yields than the random walk. Key for obtaining this result is the negative correlation between the credit spread forecast error and the forecast error on German government bond yields. Irrespective of the forecasting model for credit spreads, the correlation is consistently negative around -0.5 in magnitude.

Two aspects of results presented in this paper are relevant for government debt management. First, the fact that our "sum-of-the-parts" model can outperform a naive benchmark indicates an active role for the debt management office in managing the duration of the government debt portfolio. Second, the negative correlation of forecast errors could be reflected in the simulation model for optimizing the interest rate costs through the negative correlation between shocks to future path of the policy rate and shocks to the sovereign credit spread. The negative correlation is intuitive as rising sovereign credit spreads tend to be offset by monetary policy easing.

The remainder of the paper is organized as follows. The next section provides a brief overview of the literature on forecasting the term structure of interest rates. Section 3 discusses the data. Section 4 presents the forecasting models and related metrics that we use to evaluate them. Section 5 discusses the results of our forecasting exercises. Finally, Section 6 concludes.

2 Related literature

Our work is related to several strands of literature on forecasting interest rates.

First, the interest in forecasting interest rates originated from evaluating the expectations hypothesis of interest rates, which postulates that the risk premiums in yields are either non-existent or constant. Paired with the rational expectations assumption, the expectation hypothesis can be tested by evaluating the predictability of yields, see e.g. Fama and Bliss (1987), Campbell and Shiller (1991). More recently, Cieslak and Povala (2015) propose a decomposition of the yield curve into the expectations hypothesis term and the risk premium, specifically into inflation expectations and a set of maturity-specific cycles. Further by controlling for the expectations hypothesis term in yield cycles, it is possible to extract a return-forecasting factor which predicts bond excess returns across all maturities. Thus, the decomposition delivers three factors with a direct link to economic quantities: expected inflation, the real rate, and the risk premium.

Second area of literature is more statistical in nature and is concerned with how to use the information in the whole yield curve to forecast interest rates. To do this efficiently, one needs to reduce the dimensionality of yields, which is often achieved through principal components or dynamic factor models, e.g. Diebold and Li (2006). Numerous dynamic factor models build on the pioneering research of Nelson and Siegel (1987) who introduced a three-factor model to fit the term structure by applying flexible, smooth parametric functions. Their model is capable of capturing most of the variation in the yield curve over time by using a small number of parameters. Svensson (1994) extended this model by introducing the fourth factor to allow for more flexibility. Diebold and Li (2006) allow the three parameters in the Nelson-Siegel framework to evolve dynamically and estimate an autoregressive model for them. The dynamic Nelson-Siegel produces term structure forecasts that perform well, especially at long horizons. Due to its parsimony combined with the fact that the model has become an important benchmark in the yield curve forecasting literature, dynamic Nelson-Siegel model is one of the models we use to produce forecasts of Slovak government bond yields. Related to the dynamic Nelson-Siegel approach is reducing the dimensionality of yields through principal component analysis (PCA). Litterman and Scheinkman (1991b) show that the first three principal components capture virtually all the variation in yields. The main issue with both the dynamic Nelson-Siegel model and principal components is that they do not utilize economic restrictions reflected in the yield curve such as the no-arbitrage condition or the separation of risk premiums and short rate expectations. As a result, it is complicated to locate the source of predictability of interest rates–a piece of information of great importance for policy makers and investors. To partially address this drawback a number of papers including Ang and Piazzesi (2003b), de Pooter et al. (2010) have extended models based on unobservable factors by adding macroeconomic variables. de Pooter et al. (2010) show that adding macro variables improves forecasting performance mostly around recession periods while models without macro variables do particularly well in low-volatility periods, such as the post-Volcker period. Given the fact that the yield curve shall reflect all available information, including macroeconomic variables, one should have a compelling argument for adding non-yield curve information to the list of forecasting variables, see e.g. Duffee (2012c) for more details.

Finally, a number of papers study the forecasting of sovereign credit spreads. This literature has historically been mostly focused on emerging market bonds and particularly on the relative importance of country-specific fundamentals and global financial variables such as the VIX index or the slope of the U.S. yield curve, see e.g. Cornelli (2012), Amstad et al. (2016). The euro zone sovereign debt crisis has redirected investors' attention to developed market bonds, e.g. Ejsing et al. (2015).

In our forecasting model for the term structure of sovereign credit spreads, we rely on a simple model that includes an autoregressive term and commonly used global financial variables.

3 Data

This section describes the data used in the paper. Our choice of the sample period for the forecasting exercise is driven by the availability of data on Slovak government bond yields.

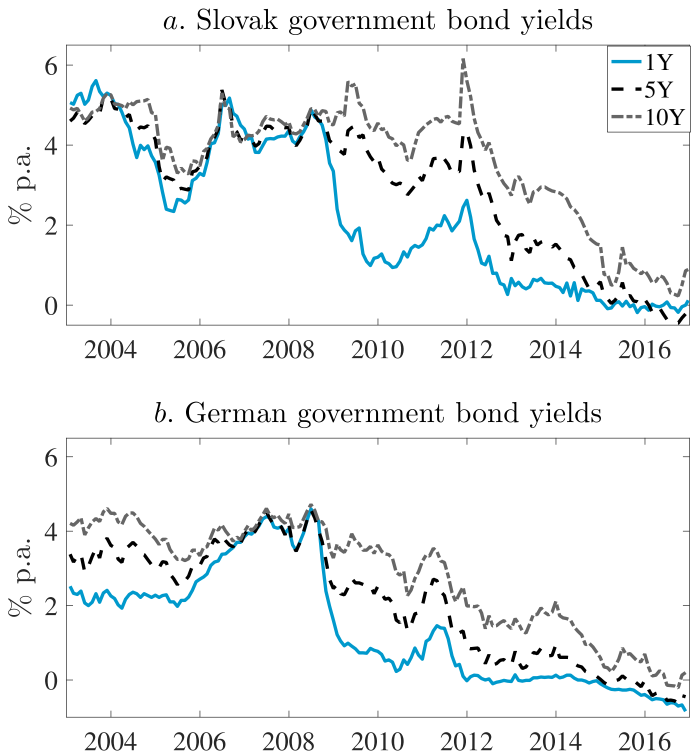
Yields

We study Slovak government bond yields in the period from January 2003 through December 2016 at a monthly frequency. The start of the sample period is determined by the availability of zero-coupon yield data. The out-of-sample period is January 2009 through December 2016. We source the zero-coupon yields on Slovak government bonds from the Ministry of Finance[1] for maturities between one and ten years. German zero-coupon government bond yields are obtained from Deutsche Bundesbank.[2] To explore the impact of regime changes on forecasting performance, we consider the following out-of-sample periods: *(i)* January 2003 through December 2016; *(ii)* January 2003 through December 2011 to assess the impact of the effective lower bound, and *(iii)* January 2009 through December 2016 to match the out-of-sample period of Slovak bond yields. The start of the sample for German yields is January 1991, which is chosen to avoid a potential regime shift due to German reunification. Zero-coupon yields on U.S. Treasury bonds are from the Federal Reserve Board.[3] We consider the following out-of-sample periods for U.S. data: *(i)* January 2003 through December 2016; *(ii)* January 2003 through December 2008, and *(iii)* January 2009 through December 2016 to assess the forecasting performance at or close to the effective lower bound.

Panel *a* in Figure 1 shows the term structure of yields on Slovak government bonds and Panel *b* shows the German government bond yields. There are several noteworthy observations. First, the yield curve is largely flat before Slovakia joined the eurozone in January 2009. Since then, the yield curve has been consistently upward-sloping with the short-end closely following the ECB policy rate and German yields. Second, spikes in the long-term yields in our sample are associated with political uncertainty and financial crises. The spike in the first half of 2006 can be attributed to an increased uncertainty about the euro adoption in the midst of early general elections. The second spike in yields in 2008-2009 is related to the

[1] Ministry of Finance of the Slovak Republic.
[2] German Bundesbank.
[3] Federal Reserve Board.

Figure 1: Zero-coupon yields on Slovak and German government bonds, 2003-2016.



The figure shows one-, five-, and ten-year zero-coupon yield on Slovak government bonds (Panel *a*) and the corresponding German government bond yields (Panel *b*). The sample period is January 2003 through December 2016. Data are monthly.

bankruptcy of Lehman Brothers, an investment bank, and its immediate aftermath. Finally, the sharpest and the largest increase in yields to date at the end of 2011 is related to the euro zone sovereign debt crisis.

Table 1 reports descriptive statistics for Slovak and German government bond yields, Panels A and B, respectively. Both, the Slovak and the German yield curve are upward-sloping with the average slope in close to 140 basis points in both cases. Slovak government bonds are less persistent which is driven by a relatively low persistence of the spread.

Credit spreads

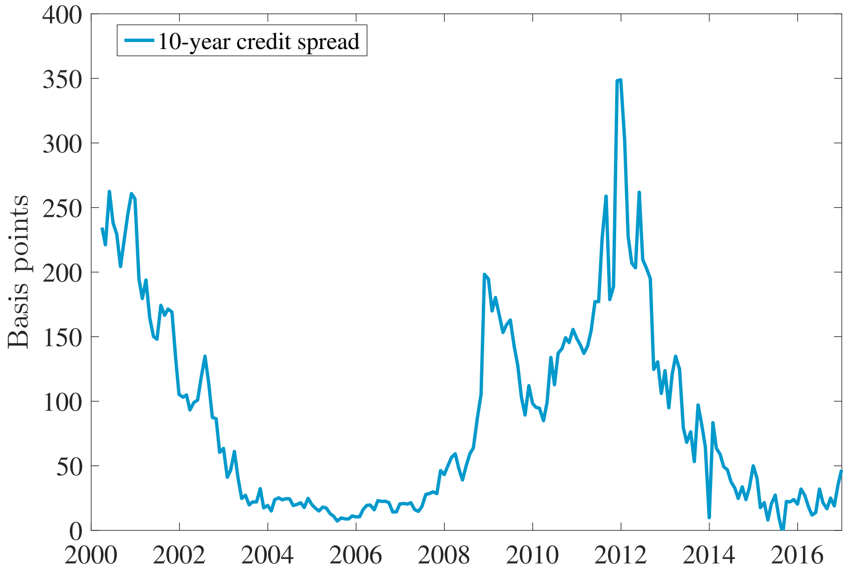
Prior to January 2009, when Slovakia adopted the euro, we use international government bonds issued by the Slovak Republic in euro to measure the credit spread

Table 1: Descriptive statistics

Panel A: Slovak yields						
	Mean	St. dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(12)$
1Y	2.27	1.83	-0.19	5.61	0.98	0.65
2Y	2.42	1.77	-0.42	5.26	0.98	0.68
3Y	2.58	1.75	-0.54	5.35	0.98	0.69
4Y	2.76	1.72	-0.55	5.38	0.98	0.69
5Y	2.93	1.69	-0.45	5.38	0.98	0.68
6Y	3.10	1.65	-0.35	5.37	0.98	0.67
7Y	3.26	1.60	-0.21	5.35	0.97	0.66
8Y	3.40	1.56	-0.06	5.59	0.97	0.65
9Y	3.53	1.52	0.09	5.91	0.97	0.64
10Y	3.65	1.48	0.23	6.16	0.97	0.63
Panel B: German yields						
	Mean	St. dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(12)$
1Y	1.37	1.54	-0.84	4.60	0.99	0.73
2Y	1.51	1.55	-0.80	4.62	0.98	0.76
3Y	1.68	1.55	-0.76	4.58	0.98	0.77
4Y	1.86	1.55	-0.69	4.56	0.98	0.77
5Y	2.04	1.54	-0.63	4.57	0.98	0.77
6Y	2.21	1.53	-0.56	4.58	0.98	0.76
7Y	2.37	1.51	-0.47	4.61	0.98	0.76
8Y	2.52	1.49	-0.38	4.65	0.98	0.75
9Y	2.66	1.47	-0.29	4.68	0.98	0.74
10Y	2.78	1.45	-0.21	4.72	0.98	0.73

We report descriptive statistics for Slovak and German government bond yields over the period January 2003 through December 2016, Panels A and B, respectively. Data are at a monthly frequency, sampled on the last trading day of each month. We present mean, standard deviation (st. dev.), minimum (min.), maximum (max.), and the autocorrelation coefficients for lags one and twelve months.

Figure 2: Sovereign credit spread, Slovak Republic



The figure shows the sovereign credit spread for debt issued by the Slovak Republic constructed from EUR-denominated international bonds. At any point in time, we select bonds that are closest to the ten-year maturity. The data are monthly. The sample period is March 2000 (issuance of the first EUR-denominated Slovak government bond) through December 2016.

to German government bond yields, see Odor and Povala (2016) for more details on the construction of the credit spread. The bonds are selected such that we cover the whole period 2003-2009 with international euro bonds whose maturity is as close as possible to the ten years. To construct the spread, we match Slovak bonds with the German coupon bonds of the same maturity.

The adoption of euro in Slovakia coincided with a sharp increase in the sovereign credit spread from below 50 basis points to above 150 basis points within several months. While the spread moved back toward the 100 basis point mark for a brief period of time, it recorded its highest level on record at around 300 basis points in the midst of the euro zone sovereign debt crisis at the end of 2011. It is worth mentioning that only in the most recent period, which coincides with the government bond purchases by the ECB, the spread reached the lows from the 2004-2007 period preceding the euro adoption.

Macro data

We obtain German all-item consumer price index (CPI) at a monthly frequency from Macrobond and construct year-on-year inflation. To account for the fact that the CPI is published with a lag (inflation for a given month is usually released in the second half of the following month), we use the CPI data that are available at the month-end. Core CPI data for the U.S. are obtained from the FRED database. The VIX index is from Bloomberg.

4 Methodology

This section outlines the metrics for evaluating interest rate forecasts and describes forecasting models. To evaluate the out-of-sample forecasting performance of competing models, we compute Root Mean Squared Forecast Errors (RMSFE) for each model and compare them to the benchmark model. RMSFE generated by model m for yield with maturity τ at horizon h is defined as:

$$RMSFE_{m,h,\tau} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}^{(\tau)}_{t+h|t,m} - y^{(\tau)}_{t+h})^2}. \quad (1)$$

Our benchmark is a naïve random walk forecast which is given by:

$$\hat{y}^{(\tau)}_{t+h|t,rw} = y^{(\tau)}_{t}, \quad (2)$$

for each maturity τ. To make the model comparison easier, we report the forecasting results as a ratio of the RMSFE of the respective model and the RMSFE of the random walk forecast. Additionally, we use the statistic proposed in Diebold and Mariano (1995) (DM) to compare the accuracy of forecasts. The null hypothesis of this statistic is that two forecasts considered have the same accuracy. The DM statistic is N(0,1) distributed under the null.

We split the forecasting exercise into two steps. First, we evaluate the predictability of German government bonds yields which serve as a benchmark for pricing Slovak government bonds. In the second step, we assess the predictability of spreads on Slovak government bonds over their German counterparts. The reason for this is the fact that these two components of Slovak government bond yields are related to different economic mechanisms and the separation allows us to better locate the source of predictability. The forecast of Slovak government bond yield for each maturity and forecasting horizon is a sum of the forecast of the corresponding German bond yield and the forecast of the credit spread. Alternatively, one could model and forecast Slovak and German yields within a multi-country reduced-form model as in Diebold et al. (2008). However, such a model would require estimating a higher number of parameters and the attribution of the predictability to various economic mechanisms would be more complicated.

We forecast the term structure of interest rates at five different horizons: one, six, twelve, 24, and 36 months. We are particularly interested in the longer horizon forecasts as these are the most relevant for debt and liquidity management.

4.1 Forecasting German government bond yields

We start with a purely statistical model of yields which is not informed by economic relationships such as the expectations hypothesis or the no-arbitrage restriction. This type of models seeks to reduce the dimensionality of yields and forecast the lower dimensional set of factors. Yield forecasts are then made with the help of estimated loadings on those factors. In general, yields at time-t should contain all necessary information for forecasting yields. Consequently, one needs to have a specific reason for adding either other than time-t yields or macro variables to the forecasting variable set, for a detailed discussion of this point see Duffee (2012c).

Dynamic Nelson-Siegel model

One of the widely used models in forecasting interest rates is the dynamic Nelson-Siegel model outlined in Diebold and Li (2006). This model describes the term structure of interest rates at each point in time as a function of a small number of parameters and time to maturity τ:

$$y_t^{(\tau)} = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + u_t^{(\tau)}, \tag{3}$$

where $y_t^{(\tau)}$ is the yield at time t with time to maturity τ and $(\beta_{i,t}, \lambda_t)$ are parameters to be determined. Following Diebold and Li (2006), we fix $\lambda_t = 0.0609$[4] and estimate $\beta_{i,t}$ by estimating equation (3) at each time t via linear regression. Once the parameters are determined, we need to forecast $\beta_{i,t+h}$ for five different forecasting horizons h. We assume AR(1) dynamics for each $\hat{\beta}_{i,t}$ and estimate them directly by regressing $\hat{\beta}_{i,t}$ on an intercept and $\hat{\beta}_{i,t-h}$:

$$\hat{\beta}_{i,t} = a_i + b_i\hat{\beta}_{i,t-h} + \varepsilon_{i,t}. \tag{4}$$

The forecast for $\beta_{i,t+h}$ is given by:

$$\hat{\beta}_{i,t+h|t} = \hat{a}_i + \hat{b}_i\hat{\beta}_{i,t}, \tag{5}$$

[4]We experimented with different values of λ, including varying λ over time. It turns out that different values of λ do not have a significant impact on forecasting performance of our model.

for $i = 1,2,3$. Estimating a multivariate VAR(1) specification for $\hat{\beta}_{i,t}$ leads to a deterioration in the out-of-sample performance of the model. The key feature of this model is a parsimonious statistical description of the yield curve that leads to a three-factor representation of yields. Note that each of the three factors is closely related to the respective principal component of yields; $\hat{\beta}_{1,t}$ is highly correlated with the first principal component of yields (level), $\hat{\beta}_{2,t}$ has a close link to the second principal component (slope), and $\hat{\beta}_{3,t}$ is closely related to the third principal component (curvature).

Forecasting the short rate and risk premiums

An alternative approach to forecasting the term structure of interest rates is to decompose the yield curve into a set economically meaningful factors which can then be predicted in the same way as in the previous section. To this end, we express an n-period yield in terms of the average of expected short rates $y_t^{(1)}$ and the risk premium $tp_t^{(n)}$:

$$y_t^{(n)} = \frac{1}{n} E_t \sum_{i=0}^{n-1} y_{t+i}^{(1)} + tp_t^{(n)}. \tag{6}$$

Early literature has focused on the short rate expectations part of (6), performing tests of the expectations hypothesis.[5] Campbell and Shiller (1991) test the expectations hypothesis using the following accounting identity that links slope of the yield curve today to the next-period's excess returns:

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \tfrac{1}{n-1}\left(y_t^{(n)} - y_t^{(1)}\right) - \frac{1}{n-1} rx_{t,t+1}^{(n)}. \tag{7}$$

Given that many forecasting models cannot beat the random walk yield forecast, the left side of (7) is close to unpredictable for long-term yields. This implies that the right hand side cannot be predictable either. However, given that slope of the yield curve varies predictably and bond excess returns are predictable as well, e.g. Cochrane and Piazzesi (2005b), it must be that the bond excess return is predictable by the slope. In a slightly modified setup, Fama and Bliss (1987) evaluate if the current term structure of interest rates contains information that is useful for forecasting changes in the short rate:

$$y_{t+n-1}^{(1)} - y_t^{(1)} = \gamma_0 + \gamma_1 \left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+n-1}, \tag{8}$$

[5] The expectations hypothesis postulates that the variation in the term structure of interest rates is solely due to changing expectations about the future path of the short rate and the risk premiums are either zero or constant.

where $f_t^{(n)}$ is defined as a contract that locks in one-period yield starting at time $t+n-1$. In a similar way, they study the predictive power of the forward-spot spread for bond excess returns:

$$rx_{t+1}^{(n)} = \delta_0 + \delta_1 \left(f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1}^{rx}, \tag{9}$$

with $rx_{t+1}^{(n)}$is defined as:

$$rx_{t+1}^{(n)} = -(n-1)\, y_{t+1}^{(n-1)} + ny_t^{(n)} - y_t^{(1)}. \tag{10}$$

Table 2 shows the regression results. As indicated in Panel A, changes in the one-period yield are highly predictable and the degree of predictability increases with horizon. While the adjusted R^2 is six per cent at a two-year horizon, it increases to 66 per cent at a five year horizon. As discussed in Fama (2006), the forward-spot spread in regression(8) captures the mean reversion of the spot rate toward its long-run mean that is most likely time-varying. The predictability of bond excess returns at an annual horizon by the forward-spot spread is reported in Panel B of Table 2. Excess returns on German government bonds are only marginally predictable by the forward-spot spread for all bond maturities as none of the loadings on the forward-spot spread is statistically significant.

The results of in-sample forecasting exercises presented above point to two sources of predictability. First, it is the local mean reversion of the short rate toward its slowly-evolving expected value. This source of predictability is most pronounced for short maturities where the effect of risk premiums is the smallest. Second source of predictability is the cyclical variation in expected excess returns which should be most visible at the long end of the yield curve. In light of results presented in Table 2, we expect the local mean reversion of the short rate to play a more important role in forecasting the term structure of interest rates.

Our next forecasting model originally proposed in Cieslak and Povala (2015) accommodates these two sources of predictability and associates the slow-moving long-run mean of yields with long-run inflation expectations.

Cieslak and Povala (2015) provide a decomposition of yields that results in three factors, each having a direct economic interpretation. In the first step, we regress yields of different maturities on a measure of long-term inflation expectations constructed from realized CPI index inflation as follows:

$$\tau_t^{CPI} = (1-v) \sum_{i=0}^{t-1} v^i \pi_{t-i}, \tag{11}$$

where $\pi_t = ln \frac{CPI_t}{CPI_{t-1}}$ is year-on-year inflation and v is parameter that determines the degree of discounting. Following Cieslak and Povala (2015), we set $v = 0.987$

Table 2: In-sample predictability of yield changes and bond excess returns of German government bonds

Panel A: Predictability of yield changes, 1991:1–2016:12				
	h=2 years	h=3 years	h=4 years	h=5 years
γ_0	-0.01	-0.01	-0.02	-0.03
	(-2.54)	(-4.23)	(-8.70)	(-15.65)
γ_1	0.56	0.92	1.12	1.28
	(1.45)	(3.54)	(5.74)	(8.41)
$\bar{R}^2$	0.06	0.22	0.44	0.66

Panel B: Predictability of excess returns, 1991:1–2016:12									
	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$	$rx^{(6)}$	$rx^{(7)}$	$rx^{(8)}$	$rx^{(9)}$	$rx^{(10)}$
δ_0	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.03	0.03
	(2.62)	(2.08)	(1.96)	(1.91)	(1.83)	(1.76)	(1.63)	(1.49)	(1.37)
δ_1	0.45	0.54	0.55	0.57	0.63	0.70	0.81	0.93	1.05
	(1.20)	(1.13)	(1.06)	(1.07)	(1.12)	(1.18)	(1.31)	(1.39)	(1.46)
$\bar{R}^2$	0.04	0.04	0.03	0.03	0.03	0.03	0.04	0.04	0.05

This table reports the results of Fama-Bliss regressions. Panel A shows the in-sample predictability of short rate changes. The regression specification is given by (8). Panel B displays the predictability of bond excess returns at an annual horizon. The predictive regression is given by (9). The sample is January 1991 through December 2016 and data are monthly. In both panels, t-statistics are obtained with Newey-West adjustment using 12 lags and are reported in parentheses.

and truncate the series at 120 months. In the second step, we assume that the one-period yield carries no risk premium and thus the residual from regressing one-period yield on τ_t^{CPI} represents the cyclical variation in the ex-ante real rate:

$$c_t^{(1)} = y_t^{(1)} - \hat{a}_1 - \hat{b}_1^{\tau} \tau_t^{CPI}. \tag{12}$$

Finally, exploiting the information in the cross-section of interest rate cycles, we construct a proxy for the variation in risk premium $\widehat{cf}_t$ by regressing $\bar{c}_t = \frac{1}{n-1} \sum_{i=2}^{n} c_t^{(i)}$ on the shortest maturity cycle $c_t^{(1)}$:

$$\widehat{cf}_t = \bar{c}_t - \hat{\gamma}_0 + \hat{\gamma}_1 c_t^{(1)}. \tag{13}$$

To forecast yields on German government bonds, we recursively construct τ_t^{CPI},$c_t^{(1)}$, and $\widehat{cf}_t$. For each forecasting horizon, we construct the corresponding yield changes $y_t^{(n)} - y_{t-h}^{(n)}$ which we then regress on $c_{t-h}^{(1)}$, and $\widehat{cf}_{t-h}$ for each maturity n:[6]

$$y_t^{(n)} - y_{t-h}^{(n)} = b_{0,n} + b_{1,n} c_{t-h}^{(1)} + b_{2,n} \widehat{cf}_{t-h} + \varepsilon_{n,t}. \tag{14}$$

Forecast for the n-period yield at horizon h $y_{t+h|t}^{(n)}$ is given by:

$$y_{t+h|t}^{(n)} = y_t^{(n)} + \hat{b}_{0,n} + \hat{b}_{1,n} c_t^{(1)} + \hat{b}_{2,n} \widehat{cf}_t. \tag{15}$$

Note that we assume random walk for τ_t^{CPI}, which is why it does not appear in the forecasting model (14). Below, we report results for (15) and its restricted version in which we leave out $\widehat{cf}_t$ given that the predictability of risk premiums in German bonds is dominated by the mean reversion in the short rate.

4.2 Predictability of spreads on Slovak government bonds

We consider three models for forecasting spreads of Slovak government bond yields. The first model is a simple random walk forecast which, similar to yields, serves as a benchmark. The second model has a simple AR(1) structure in which we estimate the following regression:

$$cs_{t+h}^{(n)} = \gamma_0 + \gamma_1 cs_t^{(n)} + \varepsilon_{t+h}^{(n)}. \tag{16}$$

Our third model is an extended version of (16) in which we add the VIX index and the slope of the German government bond yield curve:

[6] For one-year maturity, we regress yield changes only on $c_{t-h}^{(1)}$, consistent with the assumption that one-period yield is driven solely by short rate expectations.

$$cs_{t+h}^{(n)} = \gamma_0 + \gamma_1 cs_t^{(n)} + \gamma_2 VIX_t + \gamma_3 slope_t + \varepsilon_{t+h}^{(n)}. \tag{17}$$

We have experimented with other predictors such as the slope of the U.S. Treasury yield curve and settled on the model given by (17). The sample period is too short for including macro variables at a quarterly frequency.

5 Empirical results

This section discusses the results of forecasting exercises listed in the previous sections. We start with the dynamic Nelson-Siegel model which is widely used as a benchmark for forecasting the term structure of interest rates. Key in this section is the discussion about the joint behavior of forecast errors on German government bonds and Slovak credit spreads. We show that they are strongly negatively correlated which contributes the positive performance of our "sum-of-the-parts" forecasting model.

5.1 Forecasting German government bond yields

Table 3 reports the results for the dynamic Nelson-Siegel model. Panel A reports the results for the period 2003:1–2011:12, which excludes the latest episode with the policy rate at or below zero while Panel B shows the results for the full out-of-sample period 2003:1–2016:12. The results indicate that the model can predict yields better than the benchmark at forecasting horizons longer than one year and in periods when short-term yields are not at or below zero. RMSFE at the 36-month horizon in the out-of-sample period 2003:1–2011:12 are around half of the random walk RMSFE for short maturities. Including the period in which the policy rate has been at the effective lower bound (2012:1–2016:12) worsens the forecasting performance of the dynamic Nelson-Siegel model substantially bringing its RMSFEs close to the random walk benchmark. The results in Panel C (2009:1–2016:12) best illustrate the degree underperformance of the model in the low interest rate environment. The Diebold-Mariano test indicates that for almost all maturities and horizons, the random walk forecasts performs significantly better than the DNS model.

Table 4 reports the results obtained from the model that decomposes the yield curve into three economically meaningful factors. In the period excluding the effective lower bound, we observe a relatively stable forecasting performance at all horizons (Panel A) with an outperformance mostly for short maturity yields at the long forecasting horizon. While including the effective lower bound period leads to a deterioration in forecasting performace (Panels B and C), the deterioration is

Table 3: Dynamic Nelson-Siegel model

This table presents the results of out-of-sample forecasting for German government bond yields using the dynamic Nelson-Siegel model. We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naive random walk forecast. Our sample period starts in January 1991 (post-German unification) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at zero or below (January 2012 through December 2016). Panel C reports the results for the out-of-sample period January 2009 through December 2016 which matches the out-of-sample forecasting period of Slovak government bonds. Asterisks next to ratios indicate that the differrence relative to the random walk is statistically significant at one, five and ten per cent level, (***,**,*, respectively), as indicated by the Diebold-Mariano test. Data are monthly.

Panel A: Out-of-sample period 2003:1–2011:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.09*	1.08	1.13	0.84	0.47
2Y	0.99	1.05	1.18	0.87	0.49
3Y	1.01	1.06	1.23	0.92	0.52
4Y	1.05*	1.08	1.28*	0.97	0.56
5Y	1.08*	1.09	1.32*	1.02	0.62
6Y	1.08*	1.10*	1.33*	1.07	0.67
7Y	1.07**	1.10*	1.33*	1.10	0.72
8Y	1.03**	1.08*	1.32*	1.12	0.76
9Y	1.01	1.07	1.29*	1.14	0.79
10Y	1.00	1.05	1.26*	1.14	0.82
Panel B: Out-of-sample period 2003:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.11**	1.11	1.18	1.00	0.95
2Y	1.01	1.08	1.23	1.08	1.04
3Y	1.01	1.10	1.29*	1.18	1.14
4Y	1.06**	1.14*	1.34**	1.26	1.22
5Y	1.12**	1.18*	1.36**	1.31	1.28
6Y	1.14**	1.19*	1.36**	1.34	1.30
7Y	1.11**	1.17*	1.34**	1.34	1.31
8Y	1.04**	1.14*	1.30**	1.31	1.29
9Y	1.01	1.10*	1.24**	1.28	1.26
10Y	1.01	1.05	1.19*	1.23	1.23
Panel C: Out-of-sample period 2009:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.27*	1.41	1.79	2.59	3.19
2Y	1.05	1.23	1.61	2.23	2.47
3Y	1.02	1.25	1.57*	2.03	2.13*
4Y	1.11*	1.30*	1.57*	1.90	1.94*
5Y	1.22*	1.32*	1.54*	1.79*	1.80*
6Y	1.23**	1.31*	1.49*	1.69*	1.69*
7Y	1.16*	1.26*	1.43*	1.59*	1.60*
8Y	1.06*	1.20*	1.35*	1.49*	1.51*
9Y	1.00	1.13*	1.26*	1.40*	1.43*
10Y	1.01	1.06	1.18	1.32	1.36*

smaller than in the case of the DNS model. Even the out-of-sample period January 2009–December 2016, the model marginally outperforms the random walk benchmark at the long horizon for long maturities.

Importantly, the forecasting performance of this model does not deteriorate once we include the episode marked by the short-term interest rates at the effective lower bound.

The forecasting performance can be improved by restricting the model to include only the one-period cycle, see Table 5. This restricted model consistently beats the random walk benchmark at long horizons irrespective of the monetary policy environment. Not surprisingly, there is a deterioration in forecasting performance at the short end of the yield curve in the period January 2009 through December 2016 which is mainly due to the fact that the cyclical variation in short term interest rates has virtually vanished in that period. Due to its stable and good performance, we choose this model for forecasting yields on Slovak government bonds.

5.2 Predictability of spreads

Table 6 reports the results for spreads on Slovak government bonds. In Panel A, we show results from a simple AR(1) forecasting model while Panel B reports the results from the extended model. With the exception of short term maturities, both models perform worse than a naive forecast.

5.3 "Sum-of-the-parts" model of Slovak government bond yields

We combine the forecasts of German government bond yields and the forecasts of spreads on Slovak government bonds to obtain the "sum-of-the-parts" forecast of the Slovak government bond yields. Table 7 reports the results. For any of the three forecasting models of spreads, we obtain forecasting errors that are significantly lower than a naive forecast. The exception is the short end of the yield curve at long horizons where our model under-performs the random walk benchmark. This is not surprising given that short term yields have been at or close to the effective lower bound in most of the out-of-sample period. In fact, the forecast of Slovak yields at the short end is better than the forecast of short-term German yields reported in Table 3. This result suggests that credit spread and German government bond yield forecast errors interact in a way that improves the forecasting performance.

We first explore the correlation between the credit spread forecast errors and the forecast errors on German yields. Table 8 reports the results correlations for all three forecasting models for credit spreads. Irrespective of the model, credit spread forecast errors are strongly negatively correlated with the German yield forecast errors. Hence, negative correlation of forecast errors is the main source of the improvement in forecasting performance of the "sum-of-the-parts" model of Slovak government bond yields relative to German yields. The intuition for this is as

Table 4: Forecasting the short rate and the risk premium, full version

This table presents the results of out-of-sample forecasting for German government bond yields using the three-factor decomposition outlined in Section 4.1. Results in both panels are from the unrestricted version of equation (15). We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naive random walk forecast. Our burn-in sample period starts in January 1991 (post-German unification) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at zero or below (January 2012 through December 2016). Panel C reports the results for the out-of-sample period January 2009 through December 2016 which matches the out-of-sample period for Slovak government bond yields. Asterisks next to ratios indicate that the differrence relative to the random walk is statistically significant at one, five and ten per cent level, (***,**,*, respectively), as indicated by the Diebold-Mariano test. Data are monthly.

	Panel A: Out-of-sample period 2003:1–2011:12				
	1-month	6-month	12-month	24-month	36-month
1Y	1.00	1.00	1.00	0.90	0.76
2Y	1.01	1.01	1.03	0.95	0.77
3Y	1.01	1.00	1.03	0.98	0.82
4Y	1.01	0.99	1.03	1.00	0.87
5Y	1.00	0.98	1.02	1.01	0.92
6Y	1.00	0.97	1.01	1.01	0.97
7Y	1.00	0.96	0.99	1.00	1.00
8Y	1.00	0.95	0.97	0.98	1.03
9Y	1.00	0.94	0.94	0.96	1.04
10Y	1.00	0.93	0.92	0.94	1.05
	Panel B: Out-of-sample period 2003:1–2016:12				
	1-month	6-month	12-month	24-month	36-month
1Y	1.00	1.00	1.01	1.00	0.96
2Y	1.01	1.01	1.05	1.06	0.93
3Y	1.01	1.01	1.11	1.14	0.97
4Y	1.00	1.02	1.15	1.19	0.99
5Y	1.00	1.03	1.18	1.22	1.00
6Y	1.00	1.03	1.19	1.23	1.00
7Y	1.00	1.04	1.20	1.23	0.99
8Y	1.00	1.04	1.19	1.23	0.98
9Y	1.00	1.04	1.19	1.23	0.97
10Y	1.00	1.05	1.18	1.22	0.96
	Panel C: Out-of-sample period 2009:1–2016:12				
	1-month	6-month	12-month	24-month	36-month
1Y	0.99	0.99	1.11	1.98*	2.37*
2Y	1.01	1.02	1.17	1.73*	1.57
3Y	1.00	1.04	1.27*	1.65*	1.33
4Y	1.00	1.06	1.32*	1.57*	1.18*
5Y	1.00	1.08	1.33*	1.49*	1.09*
6Y	1.00	1.08	1.33*	1.44	1.02
7Y	1.00	1.09	1.32	1.40	0.98
8Y	1.00	1.09	1.31	1.37	0.95
9Y	1.00	1.09	1.30	1.35	0.94
10Y	1.01	1.09	1.28	1.33	0.92

Table 5: Forecasting the short rate and the risk premium, restricted version

This table presents the results of out-of-sample forecasting for German government bond yields using the three-factor decomposition outlined in Section 4.1. Results in both panels are from the restricted version of equation (15) in which we leave out $\widehat{cf_t}$. We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naive random walk forecast. Our burn-in sample period starts in January 1991 (post-German unification) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at zero or below (January 2012 through December 2016). Panel C reports the results for the out-of-sample period January 2009 through December 2016 which matches the out-of-sample period for Slovak government bond yields. Asterisks next to ratios indicate that the differrence relative to the random walk is statistically significant at one, five and ten per cent level, (***,**,*, respectively), as indicated by the Diebold-Mariano test. Data are monthly.

Panel A: Out-of-sample period 2003:1–2011:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.00	1.00	1.00	0.90	0.76
2Y	1.00	1.00	1.01	0.91	0.77
3Y	1.00	1.00	1.02	0.92	0.79
4Y	1.00	1.00	1.03	0.93	0.81
5Y	1.00	1.01	1.04	0.94	0.83
6Y	1.00	1.01	1.04	0.95	0.86
7Y	1.00	1.01	1.04	0.96	0.88
8Y	1.00	1.01	1.05	0.97	0.91
9Y	1.00	1.01	1.05	0.97	0.94
10Y	1.00	1.01	1.05	0.98	0.97
Panel B: Out-of-sample period 2003:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.00	1.00	1.01	1.00	0.96
2Y	1.00	1.00	1.02	1.01	0.97
3Y	1.00	1.00	1.02	1.01	0.96
4Y	1.00	1.01	1.02	1.00	0.94
5Y	1.00	1.00	1.01	0.98	0.91
6Y	1.00	1.00	1.00	0.96	0.88
7Y	1.00	1.00	0.99	0.93	0.86
8Y	1.00	1.00	0.98	0.91	0.83
9Y	1.00	1.00	0.98	0.89	0.81
10Y	1.00	1.00	0.97	0.87	0.79
Panel C: Out-of-sample period 2009:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	0.99	0.99	1.11	1.98*	2.37*
2Y	0.99	1.02	1.14	1.66*	1.73*
3Y	1.00	1.03	1.11	1.41	1.37
4Y	1.00	1.03	1.07	1.22	1.15
5Y	1.00	1.02	1.04	1.09	1.01
6Y	1.00	1.01	1.00	1.00	0.91
7Y	1.00	1.00	0.98	0.93	0.83*
8Y	1.00	0.99	0.95	0.87	0.78*
9Y	1.00	0.99	0.94	0.83*	0.73*
10Y	1.00	0.98	0.92	0.79*	0.70*

Table 6: Predictability of spreads on Slovak government bonds

This table presents the out-of-sample forecasts for spreads on Slovak government bonds to German government bonds using an AR(1) model (Panel A) and an extended model that includes the lagged spread, VIX index and slope of the German government bond yield curve (Panel B). We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naive random walk forecast. The out-of-sample period is January 2009 through December 2016. Data are monthly.

Panel A: AR(1) model					
	1-month	6-month	12-month	24-month	36-month
1Y	0.99	0.91	0.87	0.73	0.79
2Y	1.03	1.05	1.06	0.95	0.98
3Y	1.04	1.08	1.10	1.08	1.12
4Y	1.03	1.08	1.09	1.14	1.24
5Y	1.03	1.08	1.08	1.18	1.34
6Y	1.03	1.07	1.07	1.20	1.44
7Y	1.03	1.08	1.08	1.22	1.52
8Y	1.03	1.08	1.10	1.24	1.59
9Y	1.04	1.09	1.13	1.25	1.63
10Y	1.04	1.11	1.17	1.27	1.64
Panel B: Extended model					
	1-month	6-month	12-month	24-month	36-month
1Y	0.99	1.06	1.30	2.41	1.19
2Y	1.03	1.10	1.18	2.32	0.85
3Y	1.04	1.11	1.15	2.20	0.91
4Y	1.03	1.12	1.12	2.10	1.09
5Y	1.02	1.12	1.11	2.02	1.27
6Y	1.02	1.13	1.10	1.94	1.39
7Y	1.02	1.15	1.10	1.87	1.48
8Y	1.03	1.16	1.10	1.81	1.53
9Y	1.03	1.19	1.12	1.75	1.54
10Y	1.04	1.23	1.16	1.70	1.52

Table 7: Predictability of Slovak government bond yields

This table presents the out-of-sample forecasts of Slovak government bond yields. In each panel, we obtain forecasts of German bond yields from the restricted version of the three factor model that predicts changes in the short rate. Forecasts of the spreads on Slovak government bonds in Panel A are from a naive model (random walk). In Panel B, we show the forecasts of spread from an AR(1) model. Finally, Panel C reports the spread forecasts from a model that includes the AR(1) term, the VIX index and the slope of the German government bond yield curve. We consider five forecasting horizons ranging between one and 36 months. Each panel reports RMSFE ratios relative to the naive random walk forecast. The out-of-sample period is January 2009 (euro adoption) through December 2016. Asterisks next to ratios indicate that the differrence relative to the random walk is statistically significant at one, five and ten per cent level, (***,**,*, respectively), as indicated by the Diebold-Mariano test. Data are monthly.

Panel A: Restr. cycle model (German yields) + RW (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	0.66	0.70	0.86	1.18*	1.58*
2Y	0.69	0.74	0.89	1.13*	1.25*
3Y	0.70	0.75	0.87	1.02*	1.07*
4Y	0.71	0.75	0.84	0.92*	0.95
5Y	0.71	0.74	0.81	0.84	0.87
6Y	0.72	0.74	0.79	0.79	0.81
7Y	0.73	0.74	0.77	0.74	0.77*
8Y	0.73	0.74	0.76	0.71	0.73*
9Y	0.74	0.74	0.76	0.69	0.71*
10Y	0.75	0.75	0.75	0.67	0.69*
Panel B: Restr. cycle model (German yields) + AR(1) (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	0.65	0.59	0.73	1.02	1.56*
2Y	0.70	0.69	0.79	1.01	1.26*
3Y	0.71	0.72	0.77	0.91	1.12*
4Y	0.72	0.73	0.74	0.83	1.05*
5Y	0.72	0.74	0.70	0.77	1.01
6Y	0.73	0.74	0.68	0.73	0.99
7Y	0.74	0.74	0.67	0.71	0.98
8Y	0.75	0.75	0.68	0.71	0.98
9Y	0.76	0.76	0.69	0.72	0.98
10Y	0.77	0.78	0.72	0.73	0.97
Panel C: Restr. cycle model (German yields) + ext. model (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	0.65	0.80	1.15	1.37	1.70*
2Y	0.69	0.74	1.06	1.23	1.14
3Y	0.69	0.70	0.96	1.10	0.85
4Y	0.70	0.67	0.88	1.01	0.74
5Y	0.70	0.66	0.82	0.96	0.70
6Y	0.71	0.67	0.77	0.91	0.71
7Y	0.72	0.68	0.74	0.88	0.72
8Y	0.73	0.71	0.73	0.86	0.72
9Y	0.75	0.74	0.73	0.84	0.72
10Y	0.76	0.78	0.74	0.82	0.70

follows. If the forecasting model for the term structure of German interest rates underpredicts future yields this is offset by an overprediction of the corresponding credit spread with the resulting effect being the lower forecast error for Slovak government bond yields. There are several channels through which this effect can arise. First, a positive shock to the credit spread can coincide with a negative shock to the expected path of the short rate (monetary policy easing). Second, a shock to the credit spread can be negatively correlated with shocks to term premiums.

For comparison Table 9 presents the results obtained with the dynamic Nelson-Siegel model applied directly to Slovak government bond yields. The results show that the model performs poorly across horizons and maturities. In no case the DNS model delivers smaller prediction errors than the naive forecast. In fact, in some instances the worse performace is statistically significant as indicated by the Diebold-Mariano test. These results suggest that explicitly acknowledging different economic mechanisms in the forecasting process leads to an improvement in forecasting performance.

6 Conclusions

We study the predictability of Slovak government bond yields in the period January 2009 through December 2016. We document that the "sum-of-the-parts" forecasting model consistently outperforms the random walk benchmark out-of-sample. The main source of outperformance relative to the random walk benchmark is the fact that credit spread forecast errors and the forecast errors on German government bond yields are strongly negatively correlated. There are several ways in which the analysis presented in this paper can be taken further. First, it is important to better understand the economic mechanism behind the negative correlation. Second, we could incorporate survey data in our forecasting exercise.

Table 8: Correlation of forecast errors

This table presents the correlations of credit spread forecast errors with forecast errors of German government bond yields. In each panel, we obtain forecasts of German bond yields from the restricted version of the three factor model that predicts changes in the short rate. Forecasts of the spreads on Slovak government bonds in Panel A are from a naive model (random walk). In Panel B, we show the forecasts of spread from an AR(1) model. Finally, Panel C reports the spread forecasts from a model that includes the AR(1) term, the VIX index and the slope of the German government bond yield curve. We consider five forecasting horizons ranging between one and 36 months. The out-of-sample period is January 2009 (euro adoption) through December 2016. Data are monthly.

Panel A: Restr. cycle model (German yields) + RW (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	-0.46	-0.48	-0.13	-0.58	-0.58
2Y	-0.49	-0.54	-0.28	-0.59	-0.56
3Y	-0.48	-0.53	-0.35	-0.64	-0.57
4Y	-0.45	-0.52	-0.38	-0.70	-0.60
5Y	-0.40	-0.50	-0.38	-0.73	-0.62
6Y	-0.35	-0.47	-0.37	-0.74	-0.62
7Y	-0.31	-0.44	-0.35	-0.74	-0.60
8Y	-0.27	-0.40	-0.32	-0.72	-0.58
9Y	-0.24	-0.36	-0.28	-0.69	-0.55
10Y	-0.22	-0.34	-0.26	-0.65	-0.51
Panel B: Restr. cycle model (German yields) + AR(1) (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	-0.49	-0.59	-0.36	-0.58	-0.75
2Y	-0.49	-0.59	-0.50	-0.69	-0.76
3Y	-0.49	-0.57	-0.57	-0.72	-0.72
4Y	-0.45	-0.54	-0.59	-0.71	-0.69
5Y	-0.40	-0.50	-0.58	-0.70	-0.66
6Y	-0.35	-0.47	-0.55	-0.67	-0.62
7Y	-0.31	-0.43	-0.52	-0.64	-0.59
8Y	-0.27	-0.38	-0.47	-0.61	-0.55
9Y	-0.23	-0.34	-0.42	-0.57	-0.51
10Y	-0.21	-0.30	-0.38	-0.54	-0.47
Panel C: Restr. cycle model (German yields) + AR(1)-Extended model (spreads)					
	1-month	6-month	12-month	24-month	36-month
1Y	-0.48	-0.42	-0.09	-0.32	-0.08
2Y	-0.50	-0.58	-0.24	-0.50	-0.56
3Y	-0.50	-0.62	-0.35	-0.59	-0.82
4Y	-0.47	-0.61	-0.40	-0.63	-0.85
5Y	-0.42	-0.59	-0.42	-0.64	-0.83
6Y	-0.37	-0.56	-0.41	-0.63	-0.79
7Y	-0.32	-0.51	-0.40	-0.61	-0.76
8Y	-0.28	-0.46	-0.38	-0.58	-0.72
9Y	-0.24	-0.40	-0.36	-0.55	-0.68
10Y	-0.21	-0.35	-0.33	-0.51	-0.63

Table 9: Predictability of Slovak government bond yields–Dynamic Nelson-Siegel model

	1-month	6-month	12-month	24-month	36-month
1Y	1.48*	1.90*	2.47	2.24*	2.79*
2Y	1.16	1.62	2.05*	1.87*	2.09*
3Y	1.27	1.57	1.86*	1.72*	1.80*
4Y	1.43	1.59	1.77*	1.64*	1.64*
5Y	1.47*	1.59	1.70*	1.57*	1.54
6Y	1.39	1.53	1.63	1.51	1.45
7Y	1.26	1.45	1.56	1.45	1.37
8Y	1.15	1.37	1.48	1.40	1.31
9Y	1.07	1.28	1.42	1.35	1.25
10Y	1.05	1.22	1.36	1.30	1.20

This table presents the out-of-sample forecasts of Slovak government bond yields obtained from the dynamic Nelson-Siegel model. We consider five forecasting horizons ranging between one and 36 months. We report RMSFE ratios relative to the naive random walk forecast. Stars next to ratios indicate that the difference relative to the random walk is statistically significant at five per cent level, as indicated by the Diebold-Mariano test. The out-of-sample period is January 2009 (euro adoption) through December 2016. Data are monthly.

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Appendix

Forecasting U.S. Treasury bond yields

This section presents the results for U.S. Treasury bond yields. To ensure the comparability, results below are obtain in the same sample as the German yields, i.e. burn-in period starts in January 1991 and the out-of-sample period starts in January 2003.

Table (10) reports the results obtained from the dynamic Nelson-Siegel model. While the model performs well at longer horizons (one year and longer) in the period that excludes the episode with short-term interest rates at the effective lower bound, its performance deteriorates substantially in the full out-of-sample period. This is in line with the results for German yields presented in the body of the paper.

Table (11) reports the results of a forecasting exercise for U.S. Treasury bonds using the three-factor decomposition outlined in Section 4.1. In this table, we report the unrestricted version of equation (15). Similar to the DNS model, the effective lower bound period causes a significant deterioration in the forecasting performance, albeit less severe than in the case of the DNS model.

Table (12) shows the results from the restricted version of equation (15). The restricted version of the three-factor trend-cycle model performs slightly better than the unrestricted but still fails to meaningfully outperform the random walk benchmark.

Table 10: Dynamic Nelson-Siegel model, U.S. yields

This table presents the results of out-of-sample forecasting for U.S. Treasury yields using the dynamic Nelson-Siegel model. We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naïve random walk forecast. Our sample period starts in January 1991 (to make the results comparable to German yields) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at zero or below (January 2009 through December 2016). Data are monthly.

Panel A: Out-of-sample period 2003:1–2008:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.08	1.05	0.90	0.56	0.26
2Y	0.97	1.04	0.90	0.56	0.32
3Y	1.05	1.05	0.94	0.58	0.44
4Y	1.05	1.05	0.97	0.61	0.53
5Y	1.03	1.05	1.00	0.65	0.61
6Y	1.05	1.06	1.04	0.70	0.69
7Y	1.04	1.06	1.07	0.75	0.77
8Y	1.02	1.05	1.09	0.81	0.86
9Y	1.00	1.04	1.09	0.87	0.95
10Y	1.00	1.02	1.10	0.93	1.04
Panel B: Out-of-sample period 2003:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.15	1.23	1.20	0.99	0.95
2Y	1.00	1.19	1.24	1.07	1.07
3Y	1.05	1.21	1.34	1.17	1.18
4Y	1.05	1.21	1.39	1.25	1.26
5Y	1.05	1.20	1.40	1.29	1.32
6Y	1.06	1.18	1.39	1.30	1.36
7Y	1.04	1.15	1.36	1.30	1.38
8Y	1.02	1.12	1.31	1.29	1.39
9Y	1.01	1.09	1.27	1.27	1.39
10Y	1.02	1.07	1.23	1.25	1.38

Table 11: Forecasting the short rate and the risk premium, full version

This table presents the results of out-of-sample forecasting for U.S. Treasury yields using the three-factor decomposition outlined in Section 4.1. Results in both panels are from the unrestricted version of equation (15). We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naive random walk forecast. Our burn-in sample period starts in January 1991 (to make the results directly comparable to German yields) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at or close to zero (January 2009 through December 2016). Data are monthly.

Panel A: Out-of-sample period 2003:1–2008:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.01	1.01	0.94	0.67	0.29
2Y	1.01	1.04	1.05	0.70	0.28
3Y	1.01	1.02	1.07	0.71	0.32
4Y	1.01	1.01	1.10	0.75	0.37
5Y	1.01	1.00	1.11	0.81	0.41
6Y	1.01	0.98	1.11	0.85	0.46
7Y	1.01	0.97	1.11	0.89	0.51
8Y	1.01	0.95	1.09	0.93	0.58
9Y	1.01	0.94	1.07	0.97	0.66
10Y	1.01	0.92	1.05	1.01	0.73
Panel B: Out-of-sample period 2003:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.01	1.04	1.10	1.08	1.06
2Y	1.01	1.13	1.37	1.21	1.04
3Y	1.00	1.12	1.43	1.21	1.06
4Y	1.00	1.09	1.41	1.18	1.06
5Y	1.00	1.05	1.36	1.16	1.06
6Y	1.00	1.03	1.31	1.13	1.07
7Y	1.00	1.02	1.26	1.10	1.07
8Y	1.00	1.00	1.21	1.07	1.07
9Y	1.00	0.99	1.17	1.04	1.06
10Y	1.00	0.99	1.15	1.01	1.05

Table 12: Forecasting the short rate and the risk premium, restricted version

This table presents the results of out-of-sample forecasting for U.S. Treasury yields using the three-factor decomposition outlined in Section 4.1. Results in both panels are from the restricted version of equation (15) in which we leave out $\widehat{cf}_t$. We consider five forecasting horizons ranging between one and 36 months. The table reports RMSFE ratios relative to the naïve random walk forecast. Our burn-in sample period starts in January 1991 (to make the results directly comparable to German yields) and the out-of-sample period starts in January 2003. Panel A excludes the period in which short-term interest rate were at or close to zero (January 2009 through December 2016). Data are monthly.

	1-month	6-month	12-month	24-month	36-month
Panel A: Out-of-sample period 2003:1–2008:12					
1Y	1.01	1.01	0.94	0.67	0.29
2Y	1.01	1.00	0.92	0.64	0.28
3Y	1.01	0.99	0.90	0.63	0.30
4Y	1.01	0.99	0.91	0.67	0.31
5Y	1.01	0.99	0.93	0.72	0.34
6Y	1.01	1.00	0.94	0.77	0.37
7Y	1.01	1.00	0.96	0.83	0.42
8Y	1.01	1.01	0.98	0.89	0.48
9Y	1.01	1.01	1.00	0.96	0.57
10Y	1.01	1.02	1.02	1.04	0.67
Panel B: Out-of-sample period 2003:1–2016:12					
	1-month	6-month	12-month	24-month	36-month
1Y	1.01	1.04	1.10	1.08	1.06
2Y	1.01	1.04	1.10	1.08	1.05
3Y	1.01	1.02	1.08	1.07	1.03
4Y	1.01	1.02	1.07	1.06	1.00
5Y	1.01	1.02	1.06	1.06	1.00
6Y	1.01	1.02	1.06	1.06	1.00
7Y	1.01	1.02	1.05	1.05	1.00
8Y	1.01	1.02	1.05	1.05	1.00
9Y	1.01	1.02	1.04	1.04	1.00
10Y	1.01	1.02	1.04	1.02	0.99